Mathematical Logic and Theorem Proving

Assignment 4 (Linear Arithmetic and Array Logic)

Total 50 marks. Weightage 8%. Due date Mon 31 May 2021

1. Use the Fourier-Motzkin elimination technique to check the feasibility of the following system of constraints. Also use the technique to find a solution if it is feasible. (*10*)

$$1 \leq x \tag{1}$$
$$x + y + z \leq 9 \tag{2}$$
$$z \leq 4 \tag{3}$$
$$z - 4y \leq 2 \tag{4}$$

2. Use the Omega Test technique to check the integer feasibility of the following system of constraints. Also use the technique to find an integer solution if there is one. (*8*)

$$2y \leq x \tag{1}$$
$$2 + x \leq 8y \tag{2}$$
$$2y \leq 3 - x \tag{3}$$

3. Describe how you would decide the truth of a sentence in First-Order linear arithmetic over reals, using the Fourier-Motzkin technique. (*10*)

4. Consider the array logic formulas below:

   (a) $(a\langle i \lhd x\rangle[j] = x) \Rightarrow (i = j)$

   (b) $(a\langle i \lhd x\rangle[j] = x) \Rightarrow (a[j] = x)$

   (c) $(a\langle i \lhd x\rangle[j] = x) \Rightarrow (i = j \vee a[j] = x)$

   (a) Tell by manual inspection whether each of the formulas above are valid. Justify your answer. (*6*)

   (b) Use the reduction to EUF to tell whether each of the above formulas are valid. (*6*)

5. Consider the program snippet below:

```
i := 0;
for (i = 0; i < 10; i++)
  a[i] := i;
// assert (0 < x < 10) => a[x] = x
```

A candidate loop invariant for this program is

$$(0 \leq i) \wedge \forall k(((0 \leq k) \wedge (k < i)) \Rightarrow (a[k] = k))$$

and one of the verification conditions we need to check for validity is:

$$\begin{array}{l}[(0 \leq i) \wedge \forall k(((0 \leq k) \wedge (k < i)) \Rightarrow (a[k] = k)) \wedge \\ a' = a\langle i \triangleleft i \rangle \ \wedge i' = i + 1] \\ \Rightarrow \forall k(((0 \leq k) \wedge (k \leq i')) \Rightarrow (a'[k] = k)).\end{array}$$

Check the validity of the above VC using the reduction to EUF+LA. You may solve the resulting EUF+LA formula by manual inspection. (*10*)