Mathematical Logic and Theorem Proving

Assignment 5 (Linear Arithmetic, Array Logic, and Nelson-Oppen

(Not to be handed in)

1. Use the Fourier-Motzkin elimination technique to check the feasibility of the following system of constraints. Also use the technique to find a solution if it is feasible.

$$1 \leq x \tag{1}$$
$$x + y + z \leq 9 \tag{2}$$
$$z \leq 4 \tag{3}$$
$$z - 4y \leq 2 \tag{4}$$

2. Use the Omega Test technique to check the integer feasibility of the following system of constraints. Also use the technique to find an integer solution if there is one.

$$2y \leq x + z \tag{1}$$
$$2 + x - 3z \leq 8y \tag{2}$$
$$2y = 3 - 2x + 5z \tag{3}$$

3. Consider the $QF(\Sigma_A)$ formulas below. For each of them say by inspection whether they are satisfiable or not. Then apply the decision procedure to check their satisfiability. (*10*)
   (a) $a\langle i \triangleleft e\rangle[j] = e \ \wedge i \neq j \ \wedge a[j] \neq e$
   (b) $i_1 = j \ \wedge \ a[j] = v_1 \ \wedge \ a\langle i_1 \triangleleft v_1\rangle\langle i_2 \triangleleft v_2\rangle[j] \neq a[j]$.

4. Consider the $APF(\Sigma_A)$ formulas below. For each of them say by inspection whether they are satisfiable or not. Then apply the decision procedure to check their satisfiability. (*10*)
   (a) $\forall i(a\langle i \triangleleft e\rangle[i] \neq e)$
   (b) $a[k] \neq b[k] \ \wedge \ \forall i(a[i] = b[i])$.

5. Consider the formula below in the union of the quantifier-free fragments of LIA and EUF:

$$1 \leq x \wedge x \leq 3 \wedge f(x) \neq f(1) \wedge f(x) \neq f(3) \wedge f(1) \neq f(2).$$

   (a) Tell by inspection whether the formula is satisfiable or not.
   (b) Use the Nelson-Oppen technique to check satisfiability of the given formula.

6. Consider the formula below in the union of the quantifier-free fragments of LIA and Basic Array Logic:

$$a[i] \geq 1 \wedge a[i] + x \leq 2 \wedge x > 0 \wedge x = i \wedge a\langle x \triangleleft 2\rangle[i] \neq 1.$$

(a) Tell by inspection whether the formula is satisfiable or not.

(b) Use the Nelson-Oppen technique to check satisfiability of the given formula.