Program Analyis and Verification

Assignment 3 *(Interprocedural Analysis)*

Due date: Fri 7th Oct 2022.

Problem 1.

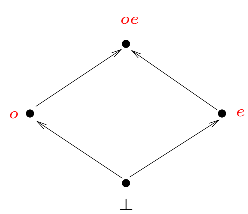
1. Consider a lattice $(D, \leq)$ and a binary operation $\star$ on D. When would you say that $\star$ is *monotonic* on this lattice?

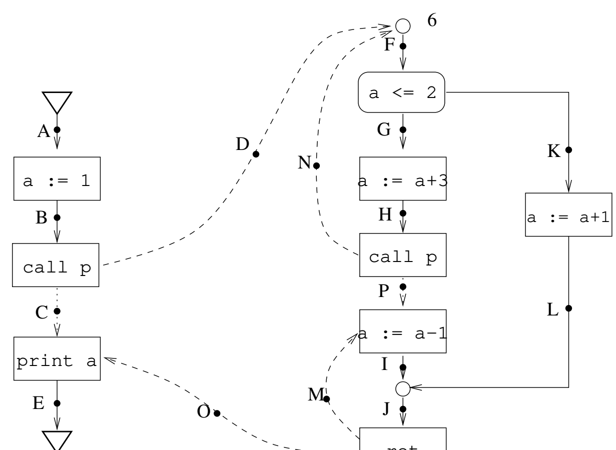
2. Let $(E, \leq)$ be a lattice. Consider the operation of function composition $\circ$ (take $f \circ g$ to mean g applied first, then f), on the lattice $(E \to E, \leq')$ of functions from E to E (with $\leq'$ being the usual pointwise ordering). Show that function composition is not monotonic w.r.t. to this lattice.

3. Would the Knaster-Tarski theorem guarantee a least solution to the equations Eq (1) in the functional approach, if we consider the lattice of all functions on the underlying domain?

(1 *pages*, 10 *marks*)

Problem 2. Consider the program below. Consider a parity analysis for the value of the variable a, using the lattice with elements $\{\bot, o, e, oe\}$ with the usual ordering as shown below. The initial abstract value is given to be oe.





Carry out an *approximate* suffix-based call-string analysis using a maximum call-string length of 2, for this program, by running Kildall's algorithm for this analysis. Give your answer in a tabular form which clearly shows each step of the iteration, as in the table below. Each row of the table corresponds to a single step of the algorithm. The last non-blank entry in each column is assumed to carry forward (so you don't have to write the entries which have not changed). The underscore denotes a "marked" value. For the call-string tables, it is enough to show only the non-$\bot$ entries.

A	B	D	E	F	G	H	I	J	K	L	M	N
$\overset{\epsilon}{\underline{oe}}$	$\overset{\epsilon}{\underline{\bot}}$	$\overset{\epsilon}{\underline{\bot}}$	$\overset{\epsilon}{\underline{\bot}}$	$\overset{\epsilon}{\underline{\bot}}$	$\overset{\epsilon}{\underline{\bot}}$	$\overset{\epsilon}{\underline{\bot}}$	$\overset{\epsilon}{\underline{\bot}}$	$\overset{\epsilon}{\underline{\bot}}$	$\overset{\epsilon}{\underline{\bot}}$	$\overset{\epsilon}{\underline{\bot}}$	$\overset{\epsilon}{\underline{\bot}}$	$\overset{\epsilon}{\underline{\bot}}$
$\overset{\epsilon}{oe}$	$\overset{\epsilon}{\underline{o}}$											
	$\overset{\epsilon}{o}$		$\overset{D}{\underline{o}}$									
...	...	...	...									

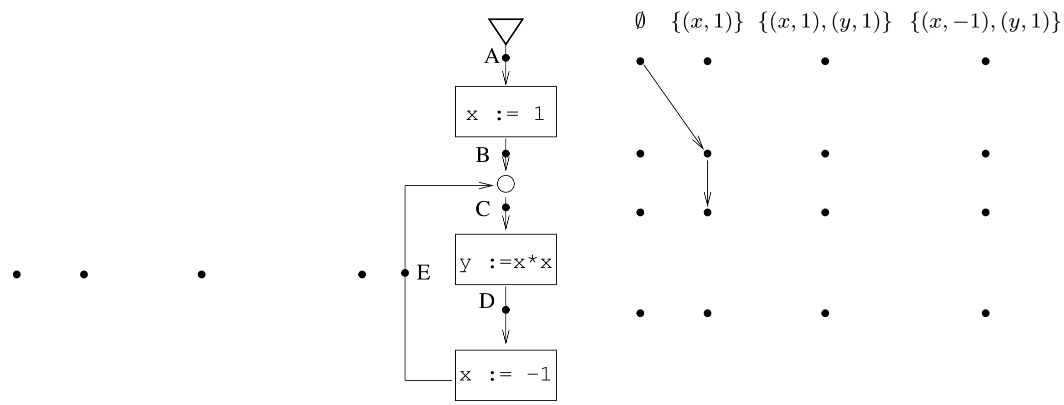
(1.5 *pages*, 10 *marks*)

Problem 3. Perform the functional approach for the above analysis on the given program, using a Kildall-style algorithm. Show the steps of the algorithm in tabular form like above, separately for the two stages (solving Eq (1) and Eq (2)). (2 *pages*, 20 *marks*)

Problem 4. Give an example program and analysis, that demonstrates that the call-string approach can be more precise than the functional approach. No need to show the working of the analysis, just give the final (approximate-)JVP values. [*Hint:* use a non-distributive analysis like constant propagation.] (1 *pages*, 10 *marks*)

Problem 5. We have seen in class that Kildall's algorithm computes only an overapproximation of the JOP for a given instance of an abstract interpretation. However, if the underlying lattice is *finite* we can give an algorithm to compute the *exact* JOP. This question asks you to suggest such an algorithm. (1.5 *pages*, 15 *marks*)

(a) Consider the program below and the Constant Propogation abstract interpretation done in class, with initial value $\emptyset$. Consider only the 4 abstract data values shown in the figure. Construct an "exploded" control-flow graph as shown below, where each program point is duplicated 4 times (one for each of the abstract values considered). Beginning with the initial value $\emptyset$ at the initial point A (we call this node in the graph "$(A, \emptyset)$"), add edges to the sucessor points by applying the transfer function for the concerned nodes. The first two steps are shown in the figure.

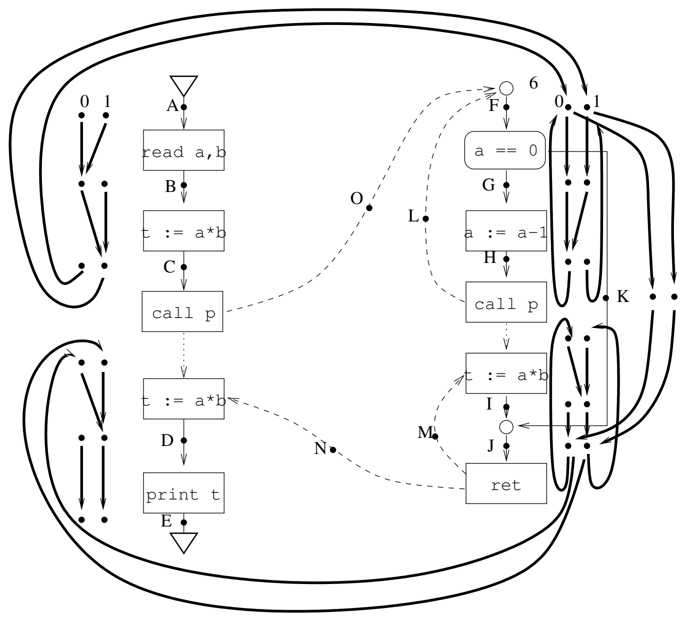


(i) Complete the procedure described above till no more edges can be added. Show the final set of edges added.

(ii) Deduce the JOP values at each point based on this exploded graph.

(b) Suggest an algorithm to compute the exact JOP for a given program P and an abstract interpretation $\mathcal{A} = (D, \leq, f_{MN}, d_0)$.

Problem 6. In a similar way to the previous problem, this problem is about a way to compute the *exact* JVP for a program with procedures. Consider the program below for which we want to do an analysis for the availability of `a*b`. We build an exploded graph representing the program and the abstract values at each program point. The edges connect an abstract data point d at program point M, to an abstract value d' at program point N, if there is a control flow edge from M to N, and $f_{MN}(d) = d'$.



Answer the following questions: (1.5 *pages*, 15 *marks*)

(a) Draw the complete exploded graph for this program and analysis (the given graph may not be accurate, so do it on your own). Take the initial data value as 0 (unavailable).

(b) By inspection, say whether the point $(D, 0)$ and $(D, 1)$ are reachable (respectively) from $(A, 0)$. Thereby infer the JVP at D.

(c) Give an algorithm to compute the JVP at each point for such a program and analysis. *Hint:* Give a way to algorithmically check reachability from a point (S, d) to another point (N, e) in this exploded graph. This is also called the "CFL Reacability" problem, where one is given a finite graph G whose edges are labelled by letters from a finite alphabet A, and a context-free language L over A (via a context-free grammar or a pushdown automaton over A), and we need to tell whether a given target vertex t in G can be reached from a source vertex s in G, via a path in G *whose labels form a string* in L.