Software Model Checking via Abstraction Refinement

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[0] Material from lectures of Aditya Nori, Daniel Kroening, Thomas Ball and Sriram Rajamani

Outline

Motivation

- Model-checking is exhaustive exploration of state

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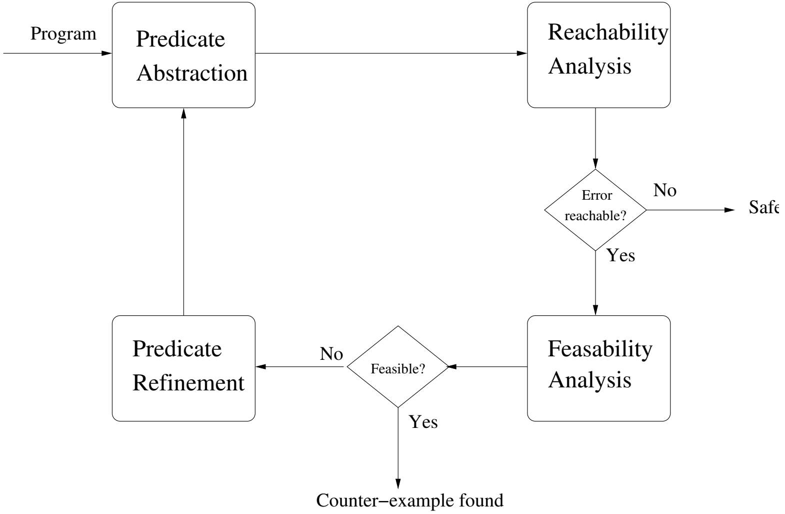
- Model-checking is exhaustive exploration of state
- Approach: Abstraction-refinement
  - **Construct an abstraction:** simple model of software having only variables and relationships important to the property to be checked
  - **Model check the abstraction:** it is easier because of smaller state space
  - **Refine the abstraction:** To reduce false errors as abstractions are over-approximations

Overview


Program
Predicate Abstraction
Reachability Analysis
Error reachable?
No
Safe
Yes
Feasability Analysis
Feasible?
No
Predicate Refinement
Yes
Counter–example found

Example


```
1.do{
2.    acquire_lock();
3.    oldx = newx;
4.    if(*){
5.        release_lock();
6.        newx++;
    }
7.} while(newx != oldx);
8.release_lock();
```

Example

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    acquire_lock();
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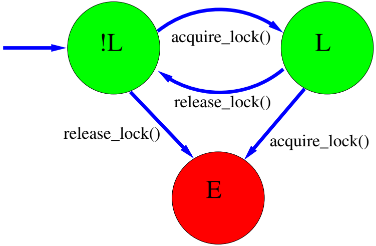
```

Does this code obey locking rule?



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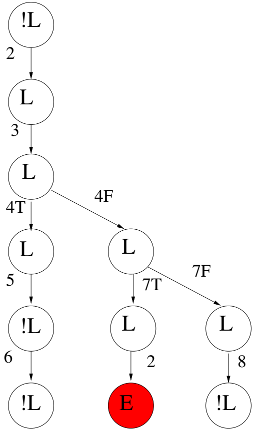

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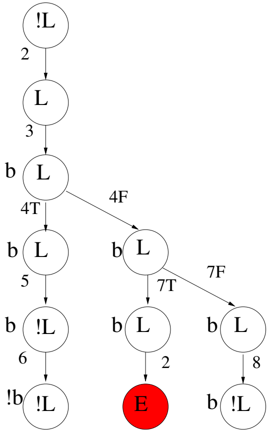
Adding b: (oldx==newx)



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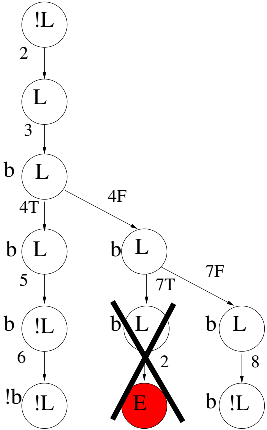

Predicate Abstraction as Abstract Domain

- Given a set of predicates $p_1, p_2, \ldots, p_n$ over set of states S
- Abstract state is valuation of these predicates:

$$s_a = \mathbb{B}^n$$

- Abstraction function is:

$$\alpha(s) = \langle p_1(s), p_2(s), \ldots, p_n(s) \rangle$$

- Transfer functions are the strongest post-conditions

Predicate Abstraction

Different ways of Predicate Abstraction

- Abstract state graphs (Graf and Saidi)
- Boolean programs (SLAM)
- Abstract Reachability Tree (BLAST)

Boolean Program

A C-like program with only Boolean variables

```
1.do{
2.    acquire_lock();
3.    oldx = newx;
4.    if(*){
5.        release_lock();
6.        newx++;
    }
7.} while(newx != oldx);
8.release_lock();
```

```
do{
    locked=true;
    b=true;
    if(*){
        locked=false;
        b=b?false:*;
    }
} while(!b);
locked=false;
```

Reachability Analysis

- We can use the idea of intersection of pushdown automata and DFA
- Pushdown automata is for call stack
- DFA is for the property we are going to check
- Other techniques like IDFS (RHS-95) can be used (to be discussed later)

Feasibility Analysis

- A path can be converted to corresponding path formula
- If formula is satisfiable $\implies$ path is feasible
- Else path is infeasible

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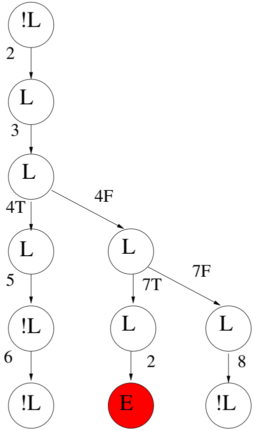

Refinement of Predicates

- Predicates are refined based on the counter-example found
- Deduce from the path formula which was unsatisfiable
- oldx=newx $\wedge$ newx!=oldx is unsatisfiable
- Add the predicate oldx=newx
- Perform all the steps again

BLAST

- **B**erkeley **L**azy **A**bstraction **S**oftware verification **T**ool
- Program is represented as a set of Control Flow Automata(CFA) for each function
- Abstract Reachability Tree (ART) is constructed for Model checking
- When ART is *complete*, BLAST terminates

Control Flow Automata(CFA)

- A CFA is a directed graph with
  - Vertices: program counter values
  - Edges: program operations
- Edges are labelled by the instruction
- An instruction can be
  - basic block of assignments
  - assume predicate
  - function call with call by value parameters
  - return instruction

Example

```
 Example() {
1:   if (*){
7:      do {
                got_lock = 0;
8:              if (*){
9:                 lock();
                   got_lock++;
                }
10:             if (got_lock){
11:                unlock();
                }
12:     } while (*)
     }
2:   do {
          lock();
          old = new;
3:        if (*){
4:           unlock();
             new++;
          }
5:   } while (new != old);
6:   unlock();
     return;
}
```

```
lock(){
   if (LOCK == 0){
      LOCK = 1;
   } else {
      ERROR
   }
}

unlock(){
   if (LOCK == 1){
      LOCK = 0;
   } else {
      ERROR
   }
}
```

[0]Example taken from Henzinger et.al., Lazy Abstraction, POPL, 2002

Example
Control Flow Automaton


```
 Example() {
1:  if (*){
7:     do {
             got_lock = 0;
8:           if (*){
9:              lock();
                got_lock++;
             }
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12:    } while (*)
    }
2:  do {
        lock();
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            new++;
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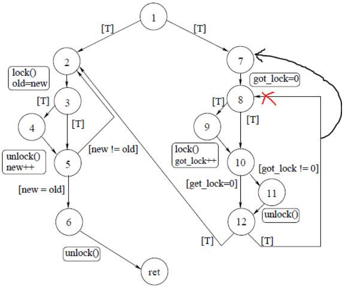
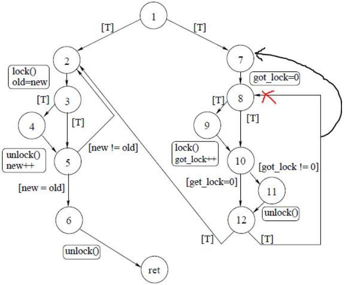


Example
Construction of Reachability Tree



Example

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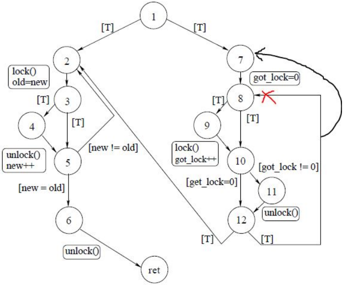

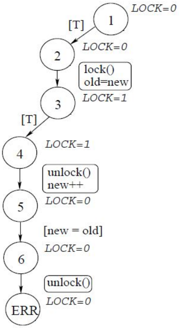

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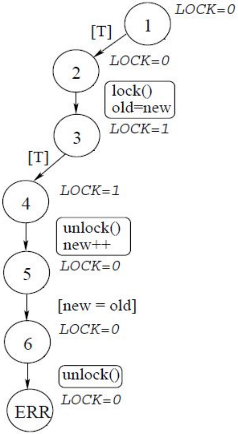

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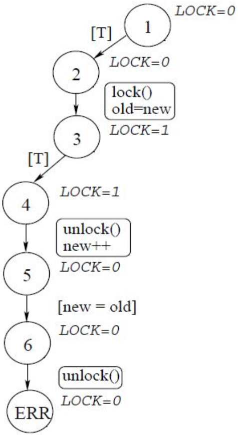

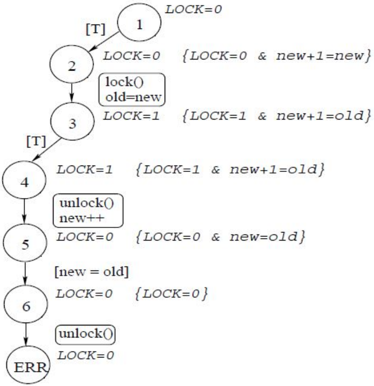

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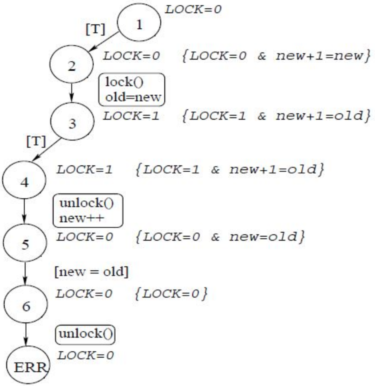

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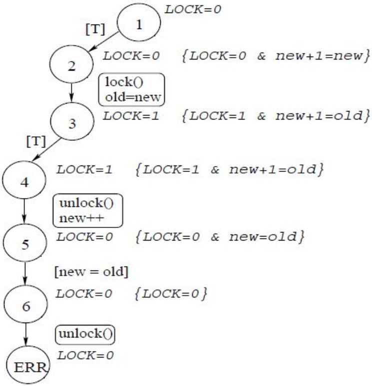

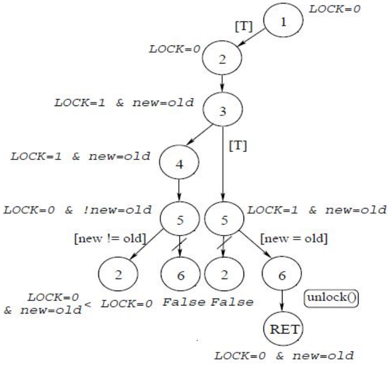

Craig's Interpolant

If $\phi_1 \wedge \phi_2 = \mathit{false}$ then there exists ψ such that

- $\phi_1 \implies \psi$
- $\psi \wedge \phi_2 = \mathit{false}$
- ψ contains variables which are common to both ϕ_1 and ϕ_2

ψ is called interpolant

Example

$$\phi_1 : x = 0 \wedge y > x$$
$$\phi_2 : y < z \wedge z < 0$$

Example

- $\phi_1 \wedge \phi_2 = \mathit{false}$

$\phi_1 : x = 0 \wedge y > x$

$\phi_2 : y < z \wedge z < 0$

Example

$\phi_1 : x = 0 \wedge y > x$
$\phi_2 : y < z \wedge z < 0$

- $\phi_1 \wedge \phi_2 = false$
- $y > 0$ is an interpolant
- $(x = 0 \wedge y > x) \implies y > 0$
- $y > 0 \wedge (y < z \wedge z < 0) = false$
- y is the only common variable in ϕ_1 and ϕ_2

Motivation

- A path can be broken in two parts, ϕ_1 and ϕ_2
- ψ_i represents a set of states somewhere in the path from where you cannot go on by taking ϕ_2.

Motivation for using Craig's Interpolation

1:	$x := ctr;$	$\langle x,1\rangle = \langle ctr,0\rangle$	$x = ctr$
2:	$ctr := ctr+1;$	$\langle ctr,1\rangle = \langle ctr,0\rangle + 1$	$x = ctr - 1$
3:	$y := ctr;$	$\langle y,2\rangle = \langle ctr,1\rangle$	$x = y - 1$
4:	$\texttt{assume}(x = m);$	$\langle x,1\rangle = \langle m,0\rangle$	$y = m + 1$
5:	$\texttt{assume}(y \neq m+1);$	$\langle y,2\rangle = \langle m,0\rangle + 1$	

Figure 2. Infeasible trace; constraints; predicates.

Proof of Unsatisfiability

Definition 1. *A* proof of unsatisfiability Π *for a set of clauses* C *is a directed acyclic graph* (V_Π, E_Π), *where* V_Π *is a set of clauses, such that*

- *for every vertex* $c \in V_\Pi$, *either*
  - $c \in C$, *and* c *is a root, or*
  - c *has exactly two predecessors,* c_1 *and* c_2, *such that* c *is the resolvent of* c_1 *and* c_2, *and*
- *the empty clause is the unique leaf.*

Craig's Interpolation

Definition 2. *Let (A, B) be a pair of clause sets and let Π be a proof of unsatisfiability of $A \cup B$, with leaf vertex* FALSE. *For all vertices $c \in V_\Pi$, let $p(c)$ be a boolean formula, such that*

- *if c is a root, then*
  - *if $c \in A$ then $p(c) = g(c)$,*
  - *else $p(c)$ is the constant* TRUE.
- *else, let c_1, c_2 be the predecessors of c and let v be their pivot variable:*
  - *if v is local to A, then $p(c) = p(c_1) \vee p(c_2)$,*
  - *else $p(c) = p(c_1) \wedge p(c_2)$.*

The Π-interpolant of (A, B), denoted $\text{ITP}(\Pi, A, B)$ *is* $p(\text{FALSE})$.