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Mechanism Design for Green, Truthful Procurement Auctions

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Abstract— **Auction based mechanisms have become popular in industrial procurement settings. These mechanisms minimize the cost of procurement and at the same time achieve desirable properties such as truthful bidding by the suppliers. In this paper, we investigate the design of truthful procurement auctions taking into account an additional important issue namely carbon emissions. In particular, we focus on the following procurement problem: A buyer wishes to source multiple units of a homogeneous item from several competing suppliers who offer volume discount bids and who also provide emission curves that specify the cost of emissions as a function of volume of supply. We assume that emission curves are reported truthfully since that information is easily verifiable through standard sources. First we formulate the volume discount procurement auction problem with emission constraints under the assumption that the suppliers are honest (that is they report production costs truthfully). Next we describe a mechanism design formulation for green procurement with strategic suppliers. Our numerical experimentation shows that emission constraints can significantly alter sourcing decisions and affect the procurement costs dramatically. To the best of our knowledge, this is the first effort in explicitly taking into account carbon emissions in planning procurement auctions.**

I. Introduction

World-wide, there has been a quite intense activity by all countries and global organizations to address the issues raised by climate change and global warming. The major factor for global warming is the emissions of greenhouse gases (GHG) into the environment. Kyoto Protocol [1] is one such international agreement that established commitments by all industrialized nations to reduce greenhouse gas (GHG) emissions. Kyoto protocol includes developed as well as developing nations each having their own responsibilities towards mitigation of GHGs. The nations have accepted targets for limiting or reducing emissions, that are expressed as levels of allowed emissions, or assigned amounts over the 2008-2012 periods. To achieve these targets, the Kyoto protocol provides three mechanisms namely clean development mechanism (CDM), Joint Implementation (JI) and emission trading [1]. The emission trading involves trading of unit of GHG at different levels starting from international trading to local trading [2]. Carbon trading is a market-based administrative approach/ mechanism designed to address the climate change concern and has led to development of international carbon market [2]. Carbon market uses the Cap and Trade mechanism. Under cap and trade system, the government sets the cap on the amount of GHG emissions on emitters. These are called the allowances. These emission allowances can be traded in carbon market. The emitting sources are generally free to buy, sell, or trade allowances among each other with the provision that each source must have sufficient allowances in its account at the end of each compliance period to cover its emissions during that period. The driving force for carbon trading is the relative cost of solutions for emission reduction between two players of the market. For example a company that can reduce carbon emission using low cost technology available to itself, can sell the excess emission rights to another company having expansive emission reduction technology. The Kyoto Protocol has made the companies realize now that they will have to pay for their emissions under business as usual strategies. Any company now will have the following options:

- Make their own activities or processes green.
- Invest in emission reduction projects under CDM, JI etc.
- Offset the emissions by buying carbon credits from the market.

Procurement processes provide opportunities for reducing carbon emissions for any company. Auction based mechanisms are extremely relevant in modern day electronic procurement systems since they enable a promising way of automating negotiations with suppliers and achieving the ideal goals of procurement efficiency and cost minimization [3]. In this paper, we are interested in auction based procurement mechanisms keeping in view the objective of limiting carbon emissions that can be attributed to the suppliers from whom the materials are sourced. In this paper, by green procurement, we mean sourcing from suppliers taking into account the emissions incurred by the suppliers. Procurement may also involve certain other sources of emissions such as due to logistics and transportation but we leave their discussion to future work.

A. Motivating Example

Assume that a buyer X wishes to procure a certain number of units of a homogeneous item from a pool of suppliers. Suppose four suppliers A, B, C, and D can supply one unit of the item and we have information on the per unit cost and per unit emissions from each of the suppliers. Let (cost per unit, number of emissions per unit) for the four suppliers be as follows.

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Supplier	Per Unit Cost	Per Unit Emission
A	200	4
B	160	6
C	150	8
D	130	10

The meaning of the above is as follows. Supplier A charges Rs 200 per unit and each unit manufactured by A contributes 4 carbon units of emissions. Suppose the buyer X wants to procure 2 units. The following table provides all six possibilities of procuring two units.

Suppliers	Total Cost	Number of Emissions
(A, B)	360	10
(A, C)	350	12
(A, D)	330	14
(B, C)	310	14
(B, D)	290	16
(C, D)	280	18

If the objective is to minimize the total cost of procurement without worrying about the emissions, then the solution would be to procure from C and D (with total cost of 280). However, if the buyer would like the total amount of emissions to be less than 15 units, then only the first four solutions in the table would be feasible and the optimal solution would be to procure from B and C (with total cost 310). We can say that the price of greenness is 30 (which is 310 minus 280).

In the above discussion, we have assumed that the bids from the suppliers are truthful. Typically, suppliers could exhibit strategic behavior and bid higher than their true willingness to sell. Also, they might report their emissions lower than the actual levels. Since emission levels depend on the technology and processes used by the suppliers and these are usually common knowledge, it is safe to assume that the emission levels are reported truthfully by the suppliers. In this setting, if we want the suppliers to truthfully reveal their willingness-to-sell values, then we will have to use mechanism design [4].

A simple mechanism that could be used for truthful reporting by the suppliers is Vickrey auction [4]. Vickrey auction has two powerful properties, namely that the allocation is optimal (in this case, minimize total cost of procurement among feasible solutions) and the mechanism is dominant strategy incentive compatible (that is, bidding truthfully is best for each supplier regardless of what the other suppliers bid). Using Vickrey auction here, the allocation will be to procure from B and C; the payments to B and C will be higher than their respective bids 160 and 150 by the corresponding Vickrey surplus. The Vickrey surplus for an allocated player is simply the marginal contribution the player makes to the cost of allocation. In this case, the cost of allocation in the presence of player B is 310 whereas the cost of allocation in the absence of player B is 330 (corresponding to the allocation A,D). Thus the marginal contribution of B to the cost of allocation is 20 and the Vickrey payment to B is 160 + 20 = 180. Similarly the Vickrey surplus of player C is also 20 and the Vickrey payment to C is 150 + 20 = 170. Thus the total payment by the buyer becomes 180 + 170 = 350, which is 40 higher than 310.

Motivated by the above example, in this paper, we investigate the problem of procuring multiple units of a single homogeneous item from a pool of suppliers in the presence of volume discount bids, satisfying (a) cost minimization (b) truthful bidding by suppliers (c) emission constraints. In the literature, only the first two criteria have been considered so far and to the best of our knowledge, this is the first time emission constraints are taken into account. We assume a fairly general way of specifying emissions as a function of number of units supplied.

B. Relevant Work

A comprehensive review of auction based procurement mechanisms appears in [3]. The volume discount auction, which is a key mechanism discussed in our current paper has been discussed in a number of papers, for example [5], [6], [7], [8]. The procurement problems from buyers and sellers viewpoint, have been studied extensively in literature [9], [10], [11], [12]. In a procurement context when a single buyer and multiple sellers who wish to exploit scale economies are present, a volume discount auction is appropriate. Here, suppliers provide bids as a function of the quantity that is being purchased. The winner determination problem for this type of auction mechanism is to select a set of winning bids where, for each bid, we select a price and quantity so that the total demand of the buyer is satisfied at minimum cost.

Green procurement is a mechanism for an organization to express its societal and environmental responsibilities. It is a concept of procuring products and services that reduces environmental impact. Green procurement can help an organization to achieve lower waste disposal costs, waste treatment costs and energy costs [13], [14], [15]. There are many popular reports available on green procurement on the Internet but they do not delve deep into how carbon emissions can be explicitly taken into account in procurement auctions.

Two recent papers have touched upon green constraints as a part of procurement modeling. Benjaafar et al. in their paper [16], consider a supply chain formation problem with carbon emissions considered as a decision parameter for operational decision-making with regard to procurement, production, and inventory management. They assume that suppliers are subject to mandatory caps on their carbon emissions. The suppliers and the firm that procures from the suppliers have to pay taxes on the amount of carbon emissions they emit above the cap. Tarek et al. [17] have formulated a green procurement problem for a typical supply chain. They have considered only carbon credit trading in their MIP (mixed integer program) formulation for meeting the carbon cap. Neither of the approaches explicitly takes into account emission costs as reported by the suppliers. Besides both the papers above assume that the suppliers are honest and report their private information truthfully. Our

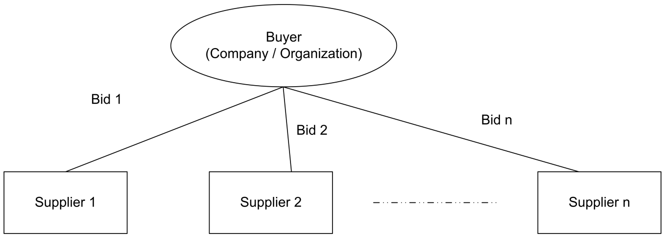

Fig. 1. Single Buyer Procurement Scenario

paper captures green constraints and also models the strategic (or game theoretic) nature of the suppliers.

C. Contributions and Outline

We start Section II with a standard formulation of the procurement auction problem as a mixed integer linear program. This formulation assumes that the suppliers report their cost curves (including volume discounts) truthfully. In fact such a formulation appears in many papers in the literature, for example [5], [3], [6], [7], [8]. We also immediately provide a formulation for making this auction truthful using fairly standard machinery in mechanism design literature.

In Section III, we present three settings of procurement auctions with green constraints. First, we present a formulation of a procurement auction with emission constraints expressed in terms of marginally decreasing piecewise linear functions. We call this green procurement auction. We then generalize this to a green, truthful procurement auction using mechanism design. Finally, we present a green procurement auction where the carbon credit offset could be bought from a carbon market.

In Section IV, we present an illustrative, stylized case study and carry out a numerical study which enables us to gain several insights. In Section V, we provide a summary and present several directions for future work in this nascent area.

II. Classical Volume Discount Procurement Auctions

For the sake of completeness, we start with a discussion of models of classical procurement auctions from [5], [3], [6], [7], [8]. We have a single buyer, who wants to purchase m homogeneous items (refer Figure 1). But these items may be supplied at different prices, because of volume discounts. Typically, the price per additional unit of the item for supply decreases as the number of items ordered from a supplier increases. Procurement auctions with volume discount is common practice in the industry. In these auctions the per unit price curve is a piecewise linear function, as illustrated on Figure 2.

The procurement problem faced by the buyer is to determine how much of the commodity to buy from each of the suppliers so as to minimize the total procurement cost. Suppliers that are chosen to deliver some amount the commodity are called winners. The buyer usually has additional business and/or operational requirements that any feasible set of trades must meet. Examples for such requirements include:

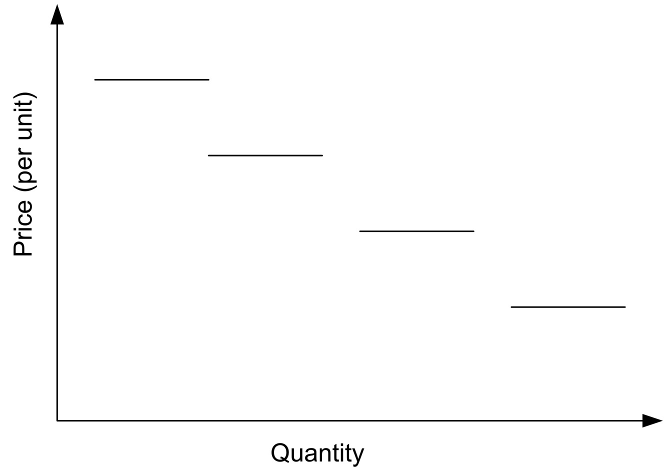

Fig. 2. Volume Discount Bid Scenario

TABLE I
NOTATION FOR VOLUME DISCOUNT AUCTIONS

Q	quantity of item
K	number of suppliers
k	index for the suppliers ($k = 1, \ldots, K$)
B_k	supply curve (bid) from supplier k
M_k	number of price-quantity pairs in bid B_k
j	index for price-quantity pairs, $j = 1, ldots, M_k$
P_{kj}	unit price the supplier charges if the number of units bought from this supplier is within the j^{th} interval $\left[Q_{kj,low}, Q_{kj,high}\right]$
x_{kj}	decision variable that takes value 1 if the buyer buys a quantity in the range $\left[Q_{kj,low}, Q_{kj,high}\right]$
z_{kj}	a continuous variable that specifies the exact number of units procured
PC_{vd}	Procurement cost using volume discount
PC_{truth}	Procurement cost using volume discount and VCG

- lower and upper limits on the number of winners so that the buyer does not rely on too few suppliers but does not increase its overhead costs by managing too many supplier accounts,
- minimum average quality requirement on winning suppliers
- measures of delivery reliability, emissions, labor practices, etc.

A. Volume Discount Auctions

Table I provides the notations for the volume discount procurement auction model described as follows [3], [6]:

- The buyer needs to procure quantity Q of an item.
- The buyer identifies a list of K potential suppliers who can bid in the auction.
- Each potential supplier submits a bid B_k composed of a supply curve. A supply curve from supplier k consists of M_k price quantity pairs: $(P_{k1}, [Q_{k1,low}, Q_{k1,high}]), \ldots, (P_{kM_k}, [Q_{kM_k,low}, Q_{kM_k,high}])$.
- Each price quantity pair $(P_{kj}, [Q_{kj,low}, Q_{kj,high}])$ specifies the price P_{kj} that a supplier k charges per unit of

the item if the number of units bought from the supplier k is within the interval $[Q_{kj,low}, Q_{kj,high}]$.

- We assume that the quantity intervals for the supply curve are all pairwise disjoint. Also, note that different unit prices are used for different ranges within the overall quantity (that is, if a quantity spans multiple intervals, the unit price for different spanned intervals will be taken as the designated unit prices in the intervals).

The MIP formulation for this problem (see Table I) is given by:

$$\min \sum_{k=1}^{K} \sum_{j=1}^{M_k} (z_{kj} P_{kj} + x_{kj} C_{kj}) \quad (1)$$

$$s.t.$$

$$z_{kj} - (Q_{kj,low} - Q_{kj,high})\, x_{kj} \quad \leq \quad 0 \;\; \forall k,\; j \quad (2)$$

$$\sum_{j=1}^{M_k} x_{kj} \leq 1 \;\; \forall\, k \quad (3)$$

$$\sum_{k=1}^{K} \sum_{j=1}^{M_k} (z_{kj} + x_{kj} Q_{kj,low}) \geq Q \quad (4)$$

$$x_{kj} \in 0, 1 \forall\, k,\; j$$

$$z_{kj} \geq 0 \forall\, k\; j$$

The coefficient C_{kj} is a constant and denotes the total price paid to supplier j in the fully sold intervals provided the amount falls into interval j:

$$C_{kj} = \sum_{l=1}^{j-1} P_{kl} \left(Q_{kl,high} - Q_{kl,low} \right) \quad (5)$$

Even with a single item, this MIP formulation is NP-hard [18]. Additional side constraints such as a limit on the number of winning suppliers and quantity constraints at the level of the supplier increase the complexity of the decision problem. We can call this procurement cost as PC_{vd}.

B. Truthful Volume Discount Auctions

We can extend the volume discount scenario by introducing strategic bidding and a standard VCG (Vickrey-Clarke-Groves) mechanism [4]. The suppliers are rational and may behave strategically. We therefore need to use a truth inducing mechanism for this scenario. We use a Vickrey based payment rule, belonging to the truth-inducing VCG family described, for example, in [4]. The rule, in words, essentially is:

Payment to vendor k	=	Bonus payment on account of the value account of the value that the vendor k adds to the system by participating in the auction	+	Bid of vendor k

This can described mathematically as follows:

In this auction mechanism, the buyer submits a consumption vector $\mathbf{q}$ to the auctioneer. Supplier k submits to the buyer (auctioneer) a bid function $\mathcal{F}_k(\mathbf{x}_k)$ for supplying $\mathbf{x}_k$ units.

Therefore, if $\pi(\mathbf{q})$ is the optimal value of the objective function for a given $\mathbf{q}$; $\mathcal{Q} = \{\mathbf{q} : \mathbf{q} > \mathbf{0}, \pi(\mathbf{q}) < \infty\}$ and we restrict $\mathbf{q} \in \mathcal{Q}$ to ensure sufficient supply capacity. If $(\mathbf{x^T}, \mathbf{y^T})$ is an optimal solution, and $\pi^{-k}(\mathbf{q})$ is the optimal value of the objective function without the supplier k, then the buyer will pay supplier k:

$$\psi_k^T(\mathbf{q}) = \pi^{-k}(\mathbf{q}) - \pi(\mathbf{q}) + \mathcal{F}_k(\mathbf{x}_k^T) \quad (6)$$

This payment rule induces rational suppliers to bid their costs irrespective of other suppliers' bids. But the auctioneer has to bear the incentives for truth telling and the procurement cost PC_{truth} (say) will be more than PC_{vd}. We can define *truth incentive cost* as the extra cost $PC_{truth} - PC_{vd}$ the buyer has to pay to ensure truth telling.

TABLE II
NOTATION FOR GREEN PROCUREMENT AUCTIONS

E_{kj}	unit emission by the supplier if the number of units bought from this supplier is within the j^{th} interval $\left[Q_{kj,low}, Q_{kj,high}\right]$
F_{kj}	Constant total emission by the supplier j in the fully sold intervals provided if the buyer buys a quantity in the range $\left[Q_{kj,low}, Q_{kj,high}\right]$
T	Maximum permissible carbon emission
PC_{green}	Procurement cost using volume discount and carbon emission
PC_{gt}	Procurement cost using volume discount and carbon emission and VCG
PC_{market}	Procurement cost using volume discount and carbon market

III. GREEN PROCUREMENT AUCTIONS

In this section we will explain the various *green* extensions to the procurement auction models explained in Section II.

A. Volume Discount Auction with Green Constraints

We now extend the volume discount bid scenario by introducing carbon emission caps. As earlier, we have one buyer and n suppliers who submit volume discount bids. The buyer in addition to minimizing his procurement cost also needs to ensure that the carbon emissions associated with the procured items does not exceed a specified maximum limit. Therefore we need to introduce the carbon emission constraint to the procurement auction defined for the volume discount case. Let us call this procurement cost as PC_{green}. A supply curve from supplier k will consist of M_k price, emissions, and quantity tuples: $(P_{k1}, E_{k1}, [Q_{k1,low}, Q_{k1,high}]), \ldots, (P_{kM_k}, E_{kM_k}, [Q_{kM_k,low}, Q_{kM_k,high}])$. Kindly refer Figure 3.

The total emissions have to be less than the carbon cap. This gives the following additional green constraint (see Table II:

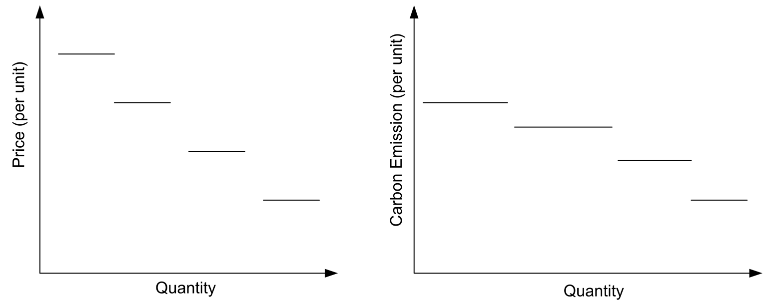

Fig. 3. Cost Curve and Emission Curve

$$\sum_{k=1}^{K}\sum_{j=1}^{M_k}(z_{kj}E_{kj} + x_{kj}F_{kj}) \;\leq\; T \tag{7}$$

$$F_{kj} = \sum_{l=1}^{j-1} E_{kl}\left(Q_{kl,high} - Q_{kl,low}\right) \tag{8}$$

Since we have introduced an additional constraint to the volume discount case, we can show that $PC_{green} \geq PC_{vd}$. Let us define *green incentive cost* as the extra cost $PC_{green} - PC_{vd}$ the buyer has to pay to ensure the carbon cap.

B. Green and Truthful Procurement Auction

We can extend the green procurement for strategic sellers again by using a standard VCG mechanism. Since the auctioneer has to bear the incentives for truth telling the procurement cost PC_{gt} (say) will be more than PC_{green} and therefore PC_{vd}. We can define *green and truth incentive cost* as the extra cost $PC_{gt} - PC_{vd}$ the buyer has to pay to ensure truth telling and the carbon cap. There are many other ways (beyond VCG mechanisms) in which a truthful mechanism could be defined, for example see Chapter 2 in [4]; a discussion of those is beyond the scope of this paper.

C. Green Procurement Auction with Carbon Market

The companies can also meet their carbon emissions cap by purchasing carbon credits from the market. We can therefore introduce carbon credit market to arrive at an alternate mechanism for the green procurement. Here we introduce carbon credit market as an additional player. We also remove the carbon cap constraint from the green procurement formulation. We can now define the objective function as:

$$Minimize \left\{ \begin{array}{l}\text{Procurement}\\ \text{Cost}\end{array} + \begin{array}{l}\text{Cost of Excess}\\ \text{Carbon Credits}\end{array} \right\}$$

where excess carbon credits to be purchased is given by:

$$\max\{0, (\text{Actual Carbon Emissions} - \text{Carbon Cap})\}$$

Let us define this procurement cost as PC_{market}. Since we are paying extra for buying the carbon credits, therefore PC_{market} will also be more than PC_{vd}. As before, we can extend this mechanism for the strategic scenario by using standard techniques from mechanism design.

To get a better idea of the mechanisms above, we now consider an illustrative case study.

IV. A Case Study

We consider that the company is interested in procuring $n = 100$ homogeneous items from its four, say A, B, C, and D suppliers. Let the total cap on emissions be 1000 units.

A. Truthful Volume Discount Auction

Let the cost curves reported by the suppliers be as follows.

Supplier	Bids
A	$< 20, [1, 10] >, < 15, [11, 20] >, < 10, [21, 50] >$
B	$< 30, [1, 20] >, < 20, [21, 30] >, < 12, [31, 60] >$
C	$< 25, [1, 20] >, < 20, [21, 40] >, < 10, [41, 70] >$
D	$< 10, [1, 20] >, < 6, [21, 40] >, < 4, [41, 50] >$

Using the standard VCG mechanism, the allocation under this setting will be 50, 0, 0, 50 for suppliers A, B, C, and D respectively. The total cost here to the buyer is $PC_{truth} = 2250$. Note that the cost curves here are the bids of the suppliers. Due to the use of VCG mechanism, these bids are truthful. VCG mechanism gives appropriate incentives for extracting truth from the suppliers. These incentives could be called the *price of truthfulness*.

B. Green and Truthful Volume Discount Auction

Let the bids of the suppliers be as follows:

Supplier	Bids
A	$< 20, 20, [1, 10] >, < 15, 15, [11, 20] >, < 10, 10, [21, 50] >$
B	$< 30, 13, [1, 20] >, < 20, 10, [21, 30] >, < 12, 7, [31, 60] >$
C	$< 25, 14, [1, 20] >, < 20, 8, [21, 40] >, < 10, 6, [41, 70] >$
D	$< 10, 30, [1, 20] >, < 6, 25, [21, 40] >, < 4, 15, [41, 50] >$

The allocation under this settings will be 0, 50, 50, 0 for suppliers A, B, C, and D, respectively. The total cost here to the buyer is $PC_{gt} = 3250$. Because of the green constraints, the cost to the buyer has increased from 2250 to 3250. This could be called the *price of greenness*.

C. Green Procurement with Carbon Credit Market

Let us assume the cost of one carbon credit (we consider all forms of carbon credit (CDM, JI etc) to be same) available in carbon market to be \$6 [19]. Here we first calculate E_{truth}

as the total emission for the products procured by considering only the truthful volume discount cost.

Considering the same example as in section IV-A, and using the bids given in section IV-B, we get $E_{truth} = 1900$ units. As the cap on emission is 1000 units, here the company has to offset the remaining $1900 - 1000 = 900$ units. To offset the carbon emissions, the company will have to buy the 900 credits from market and the cost for the same would be $900 * 6 = \$5400$. Hence the total cost of procurement would be $PC_{market} = PC_{truth} + 5400 = 7650$.

In this particular example, we can see that the company has to spend more (as $PC_{market} > PC_{gt}$ by following *business-as-usual* policy and not going for green procurement. The cost of carbon credit in the market currently is at a historic low [19]. Therefore the cost of procuring credits from the market is likely to increase. And following a green procurement policy in place of business-as-usual is recommended. However, this conclusion is only for this stylized case study and of course need not be true in general.

V. Summary and Future Work

In this paper, we have presented three settings of procurement auctions with green constraints. The first setting (green procurement auction) is a procurement auction with emission constraints expressed in terms of marginally decreasing piecewise linear functions. In the second setting, we generalized this to a green, truthful procurement auction using standard machinery in mechanism design. The third setting corresponds to a green procurement auction where the carbon credit offset is bought from a carbon market. We illustrated the mechanisms through a stylized case study.

To the best of our knowledge, this work represents the first time that both green constraints and strategic constraints are modeled as an integral part of procurement decision making. However, this is only a modest beginning in this important area. First of all, both the mathematical formulations and the numerical experimentation need to be at a more detailed level. There is a rich repertoire of challenging issues waiting to be investigated in this area taking into account green constraints: (a) procurement of heterogeneous items (b) procurement under combinatorial bidding (c) cost-minimizing auctions (d) multi-criteria auctions, etc.

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