

# Carbon Footprint Optimization - Game Theoretic Problems and Solutions

DEEPAK BAGCHI

Infosys Ltd.

and

SHANTANU BISWAS

Infosys Ltd.

and

Y. NARAHARI

Indian Institute Of Science

and

P. SURESH

Infosys Ltd.

and

L. UDAYA LAKSHMI

Indian Institute Of Science

and

N. VISWANADHAM

Indian Institute Of Science

and

S. V. SUBRAHMANYA

Infosys Ltd.

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We introduce the carbon emission reduction problem as a carbon economics problem where the countries or global industries are trying to reduce their carbon footprint at minimum cost. We discuss four problems that we have identified under the umbrella of carbon economics problems: carbon credit allocation (CCA), carbon credit buying (CCB), carbon credit selling (CCS), and carbon credit exchange (CCE). We consider the carbon credit allocation problem and provide an insight on how this problem could be addressed using optimization, game theory, and mechanism design.

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Authors' addresses:      [deepak\\_bagchi@infosys.com](mailto:deepak_bagchi@infosys.com),      [shantanu\\_biswas@infosys.com](mailto:shantanu_biswas@infosys.com),  
[hari@csa.iisc.ernet.in](mailto:hari@csa.iisc.ernet.in),      [suresh\\_p01@infosys.com](mailto:suresh_p01@infosys.com),      [udaya\\_lakshmi@csa.iisc.ernet.in](mailto:udaya_lakshmi@csa.iisc.ernet.in),  
[nv@csa.iisc.ernet.in](mailto:nv@csa.iisc.ernet.in), [subrahmanyasv@infosys.com](mailto:subrahmanyasv@infosys.com)

## 1. INTRODUCTION

World-wide, there has been a quite intense activity by all countries and global organizations to address the issues raised by climate change and global warming. A significant cause for climate change and global warming has been the green house gas (GHG) emissions by the industries across the globe. A major contributor among GHGs is the emission of carbon dioxide and hence GHG emissions are also referred as carbon emissions. Carbon emissions are measured in terms of carbon credit where one carbon credit is equal to one ton of carbon dioxide ( $CO_2$ ) emitted. Standard conversion units for other green house gases are available to do the conversion to equivalent  $CO_2$  emissions. The well known Kyoto protocol introduced the *carbon trading mechanism* to be used by global industries or organizations and to incentivize them for their efforts. The basic approach for carbon trading involves the *cap and trade* mechanism. A cap and trade system is a market based approach to controlling pollution that allows corporations or national governments to trade emissions allowances under an overall cap, or limit, on those emissions. This mechanism involves two parties, the governing body and the regulated companies or units emitting pollution. The governing body sets a limit on the total amount of  $CO_2$  and other green house gases (equated in terms of  $CO_2$ ) that could be emitted in a given period, called as *cap* and would issue rights, or allowances, corresponding to that level of emissions. After the allowances are initially distributed, entities would be free to *trade* any extra credits. Companies that can more efficiently reduce pollution sell permits to companies that cannot easily afford to reduce pollution. The companies that sell the permits are rewarded while those that purchase permits must pay for their negative impact.

### 1.1 Carbon Economics Problems

We have identified the following identified four problems in the context of a country or global industry or organization [Arava et al. 2010] (In the rest of the paper we will use the word agent to represent industry or organization):

- Carbon Credit Allocation (CCA) Problem*: Under the cap and trade mechanism, the allocation of cap becomes an important problem as limiting the carbon emissions to be less than or equal to the cap will involve cost. The allocations should consider aspects of varying cost of reductions for different agents, capacity of reduction of each agent, and policy issues. We will discuss this problem in Section 2.
- Carbon Credit Buying (CCB) Problem*: The agents that cannot reduce their carbon emissions to the level of cap can offset their carbon emissions by buying the required amount of carbon credits from the carbon market. This gives rise to an interesting problem where the company has to optimize internally and then buy the extra credits from the market keeping the procurement cost minimum.
- Carbon Credit Selling (CCS) Problem*: The agents can earn revenue by selling their carbon credits saved below the cap, to agents that produce a large volume of pollution. Thus, businesses that are involved in reducing carbon emissions or who produce low emissions in general can sell carbon credits in the market.
- Carbon Credit Exchange (CCE) Problem*: The CCB and CCS problem have

considered the situation wherein only buyers (sellers) are interested in buying (selling) carbon credits. An exchange would allow multiple buyers and sellers to trade carbon credits.

The four carbon economics problems mentioned above are essentially decision or optimization problems with incomplete information. More specifically, we have the following characteristics [Arava et al. 2010]:

- There is a set of decision makers or players who interact in a strategic way. The players have well defined payoff functions. They are rational in the sense of striving to maximize their individual payoffs. The objectives of the individual players could be conflicting. Both conflict and cooperation are involved in the interactions of the players.
- Each player has a choice of certain strategies that are available to them. The players have enough intelligence to determine their best response strategies.

A natural way of modeling problems with the above characteristics is through game theory [Narahari et al. 2009], [Arava et al. 2010]. In all the cases, it is required to implement a system-wide solution that will satisfy certain desirable properties. In order to do this, an effective way is to induce a game among the players in such a way that in an equilibrium of the induced game, the desired system wide solution is implemented. Mechanism design provides the tool for such reverse engineering of games. To explain this further, in the next section we will explore the CCA problem in more detail.

## 2. CARBON CREDIT ALLOCATION PROBLEM

We consider a global industry that has multiple divisions. Each division is an independent unit of the company or a supply chain partner and has capability to measure its carbon emissions. We assume that the industry under consideration has received a cap on its total emissions from a regulatory authority (for example, the federal government). Let  $E$  be the current total number of carbon units emitted by the industry and the cap prescribed is  $C$  and usually we have  $C < E$ . Hence the industry has to reduce or offset  $M = E - C$  emission units. The industry wants to achieve this by *optimally* allocating these  $M$  reduction units to its divisions. As the cost of reductions will vary among different divisions, the objective of the allocation here is to keep the cost minimum.

The industry here acts as social planner and asks each division to give its cost functions (or cost curves) for the reductions. We consider that the divisions have a finite set of solutions say  $S = \{s_1, s_2, \dots, s_m\}$ . The cost for implementing the solutions and the respective number of carbon credit reductions obtained is given by the sets  $C = \{cs_1, cs_2, \dots, cs_m\}$  and  $R = \{r_1, r_2, \dots, r_m\}$ . The solutions for carbon emissions reductions can be of varying types and may use either consumable or non-consumable items. If a solution is using consumable items means that the currently used raw material is replaced by another raw material that is environment friendly but may be more expensive than the former material. Here we will have: if  $r_i < r_j$ , then  $cs_i < cs_j \forall i, j \in \{1, 2, \dots, m\}$ .

If the set  $C$  is sorted in increasing order, then the set  $R$  will also be in increasing order. For consumable items, the cost can reduce with reductions if the regula-

tory authority (say government) gives a heavy subsidy on the environment friendly materials.

In the case of carbon reduction solutions using non-consumable items, we assume that the solutions are to be implemented in the order given in the set and we have  $\forall i, j \in \{1, 2, \dots, m\}$  and  $s_i, s_j \in S$  and  $i < j$ , then  $r_i < r_j$  and  $cs_i < cs_j$ . Here we will also have the set  $C_R = \{\frac{cs_1}{r_1}, \frac{cs_2}{r_2}, \dots, \frac{cs_m}{r_m}\}$  to be an increasing set, where  $C_R$  is the set for cost per unit of reduction. Also, if we apply  $s_i$  and  $s_j$  in order, then the total reduction by combined solution will be given by  $r_{ij} = K(r_i + r_j)$  where  $r_{ij}$  is the total reduction obtained and  $K \geq 1$  is a constant factor. Here the cost curve will always be an increasing curve.

In some cases, the cost curve may become a constant after it reaches a certain level of emissions. We will assume that every division will have a maximum limit on the amount of emission reductions that is possible.

Under the above described settings, the social planner is faced with two types of situations:

- Honest*: Here the individual divisions reveal their true cost curves. We can formulate this problem as an optimization problem where the objective is to keep the cost of  $M$  reduction units minimum.
- Strategic*: Here the units behave strategically and would reveal their true cost curves only if it is a best response for them. In this case, the social planner has to solve the problem in two steps: (1) extract the true cost curves and (2) determine an optimal allocation keeping the cost of reductions minimum.

### 3. OUR CONTRIBUTIONS AND RESULTS

We have proposed mechanisms/algorithms for both settings discussed above in Section 2. For the *honest* case, we have suggested a solution to the carbon credit allocation problem, which provides a cost minimizing allocation of carbon credits among different emitting agents [Arava et al. 2010]. Two variants of the problems have been considered: (a) one with limited budget and (b) with unlimited budget, by making realistic assumptions on the cost curves. A greedy algorithm is designed, which uses the cost curves (bids) of each division and computes the allocation vector which is shown to be optimal. The proposed algorithms can be used by companies to make their decision in budget planning, in deciding how much to invest to meet the immediate cap, how much to invest for future planning, etc.

In [Bagchi et al. 2011], we considered the *strategic* case and proposed a mechanism that a global company may use in allocating emission reductions to its different divisions and supply chain partners towards achieving a required target in its carbon footprint reduction program. The proposed mechanism is strategy-proof and allocatively efficient. The mechanism uses either the reverse auction method or alternately an ingenious forward auction method. The allocation of carbon emission reduction generates surplus that is then redistributed to all the participating divisions using appropriate redistribution mechanisms.

### 4. CONCLUSION

Carbon credits have become highly valuable and strategic instruments of finance in the global market and it is critical for businesses to have a well thought out strategy

for carbon footprint optimization to maximize the global good of the industry. Here we have described only one problem (carbon credit allocation problem). Other immediate problems that are waiting to be formulated and solved are the carbon credit selling, carbon credit buying, and the carbon credit exchange problems. These are problems at the level of an industry but can be extended to country or world level. We have used game theory and mechanism design offer an extremely promising mathematical framework for addressing carbon economics problems.

#### REFERENCES

- ARAVA, R., BAGCHI, D., SURESH, P., SUBRAHMANYA, S. V., AND NARAHARI, Y. 2010. Optimal allocation of carbon credits to emitting agents in a carbon economy. *6th annual IEEE Conference on Automation Science and Engineering, Toronto, Canada*.
- ARAVA, R., NARAHARI, Y., BAGCHI, D. K., SURESH, P., AND S.V.SUBRAHMANYA. 2010. Mechanism design problems in carbon economics. *Journal of IISc* 90, 381–411.
- BAGCHI, D. K., LAKSHMI, U., NARAHARI, Y., SURESH, P., AND S.V.SUBRAHMANYA. 2011. Allocation of emission reduction unit to strategic divisions and partners in a global company. *Technical Report, Department of Computer Science and Automation, Indian Institute of Science, Bangalore*.
- NARAHARI, Y., GARG, D., SURI, N. R., AND PRAKASH, H. 2009. Game theoretic problems in network economics and mechanism design solutions. *Advanced Information and Knowledge Processing Series*.