Performance Analysis of Supply Chains using Queueing Models

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Abstract

Supply chains are formed out of complex interactions amongst several enterprises whose aim is to produce and transport goods so that customer desired products are sold at various retail outlets. Computing the total average cycle time for customer orders entering such a complex network of systems is an important exercise. In this paper, we present analytical models for evaluating the average lead times of make-to-order supply chains. In particular, we illustrate the use of generalized queueing networks and Clarke's method, for computing the lead times in the dynamic and static settings, respectively.

1 The Supply Chain Process: An overview

This is an important, all-encompassing process for a manufacturing company. Supply chain process (SCP) encompasses the full range of intra-company and inter-company activities beginning with raw material procurement by independent suppliers, through manufacturing and distribution, and concluding with successful delivery of the product to the retailer or at times to the customer. One can succinctly define supply chain management (SCM) as the coordination or integration of the activities/processes involved in procuring, producing, delivering and maintaining products/services to the customer who are located in geographically different places. Traditionally marketing, distribution, planning, manufacturing and the purchasing activities are performed independently with their own functional objectives. SCM is a process oriented approach to coordinating all the functional units involved in the order to delivery process. The SCP transits several organizations and each time a transition is made, logistics is involved. Also since each of the organizations is under independent control, there are interfaces between organizations and material and information flow depend on how these interfaces are managed. We define interfaces as the procedures and vehicles for transporting information and materials across functions or organizations such as negotiations, approvals (so called paper work), decision making, and finally inspection of components/assemblies etc. For example, the interface between a supplier and manufacturer involves procurement decisions, price and delivery negotiations at the strategic level and the actual order processing and delivery at the operational level. The coordination of the SCN plays a big role in the over all functioning of the SCP. In most cases, there is an integrator for the network, who could be an original equipment manufacturer, who coordinates the flow of orders and materials through out the network.

Long term issues in SCP involve location of production and inventory facilities [1, 8], choice of alliance partners such as the suppliers and distributors, and the logistics chain. The long term decisions also include choosing make to order or make to stock policies [5], degree of vertical integration, capacity decisions of various plants, amount of flexibility in each of the subsystems, etc.

The operational issues in SCP include cycle time and average inventory computations [4], dynamic scheduling [7], inventory replenishments and the like.

2 Performance Measures

Performance measures are traditionally defined for *an organization* and are typically financial in nature, such as market share and return on sales or investment. This approach is fraught with many ills. First of all, financial indicators are lagging metrics that are a result of past decisions and are too old to be useful in operational performance improvement. Secondly, most companies do business with multiple partners selling multiple products, the individual financial statements do not delineate the winners and the losers. More importantly, several organizations are involved in the product manufacture and delivery to the customers, individual financial statements do not give a complete picture of the performance of the product or the process.

Recently [9], there are efforts to define and determine non-traditional performance measures for SCP such as lead time, quality, reliable delivery, customer service, rapid product introduction, and flexible capacity. The speed of the SCP determines the delivery time in make-to-order environments. Supply chain costs and time depend on all the constituents of the supply chain. The variability of the lead time and defect rates sum up to make up the total chain variability.

Cycle time monitoring in the supply chain networks would help in reducing the inventories, establishing good supplier relationships, reducing setup times, etc. In fact, the traditional way of functional performance measurement is presents only a partial picture of the prices.

3 Lead Time Models

The total average lead time for an order entering the supply chain is a crucial performance measure. We can compute the same in the static case as well as the dynamic case at the aggregate level, using some of the models that we present in this paper. In the static case, we require information on the processing time of all the orders that are to be scheduled across the supply chain. Whereas, in the dynamic and stochastic setting, orders for end products arrive in a random fashion, and demand random processing times at various facilities.

The static lead time models presented in Section 4 are based on an earlier work by Clarke [3] on lead time computations for assembly kind of manufacturing systems. Such models can be used to arrive at the total expected supply chain lead time and its variance. This can provide the vital link between the order processing agents and the customer, especially during order acceptance stage. For instance, an idea on the mean and variance of the supply chain lead time can help one in quoting the delivery time reliably for a given customer order.

We then cast the dynamic lead time computation problem for a class of make-to-order supply chains as multi-class open generalized queueing networks and utilize the existing efficient approximation algorithms for computing the expected supply chain lead time and variance. This is the subject matter of Section 5. We conclude this paper in Section 6.

4 Static Lead Time Models for Performance Analysis

In this section, we discuss interesting applications of Clarke's results on lead time computation for assembly like supply chains. We essentially handle the lead time computation for the static order arrival and stochastic order processing case in the following manner. Consider a network of facilities inter-connected by logistics and various inter-organizational interfaces. All the facilities start processing an order only upon receiving one such and not in anticipation. In the analysis that follows, we assume that all processing times are normally distributed with mean μ_i and standard deviation σ_i at facility i. Note that if $\mu_i \geq 3\sigma_i$ then the probability of processing time going negative is less than 0.05. Since we construct the model at the aggregate level, we ignore the internal routings within a facility and scheduling policies in use. The supply chain structure has a lot of bearing on the method used to compute the net lead time. (See Section 1.6.2 for details) In this section, we consider the case of a convergent supply chain and compute the lead time.

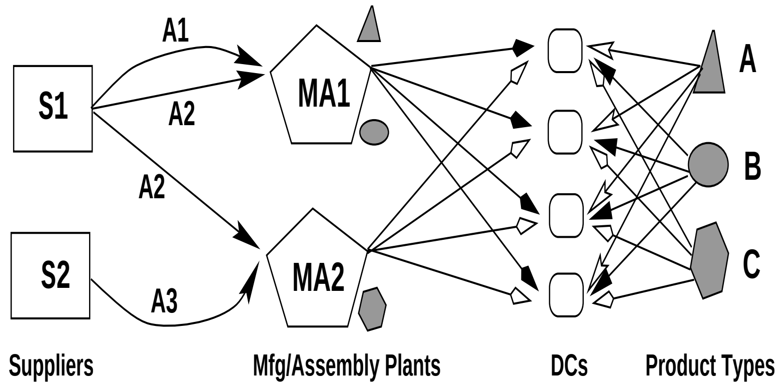

Figure 1: The supply chain network considered for analysis

4.1 Convergent Supply Chain: Lead Time Estimate for Static-Stochastic Case

Consider a supply chain network consisting of three product lines, A, B and C, which are assembled in two plants MA1 and MA2, and distributed by dedicated fleet of logistics (See Figure 1). The Bill-of-materials (BOM) for each batch of the products is: A needs one each of A1 and A2; B, one each of A1, A2 and A3; and C, one each of A2 and A3. For simplicity, we model only one hierarchy for the BOM, albeit with component commonality. Supplier S1 provides sub-assemblies A1 and A2; and S2 supplies A3. For instance, the three products could be different varieties of desk-jet printers, with the sub-assemblies representing the front panels (A1), cartridges (A3) and a base product (here, A2). We assume that the organization has worked out the relationships with two suppliers to supply the components. The distribution is handled by two separate fleet of transporters (probably third party): one for the sub-assemblies and the other for the outbound finished goods. Batch processing is assumed everywhere. We are not concerned here with optimal batch sizes and the like. For our purposes, a job represents a batch of sub-assemblies or finished products.

We decompose the network into K networks, where K is the number of job classes. In this instance, we will have to trace the path of the three classes of products A, B, and C, and obtain processing times at the various 'servers' ignoring the *interaction effects* between the various classes. These effects arise due to various scheduling policies in place, as also the alliances among supply chain members which decide the processing times. For example, let us consider product type C. The reduced network for C would be as in Figure 2. In this figure, S_1 and S_2 are the suppliers, I_3 is the interface between the suppliers and the manufacturing plant MA_2, while O_3 is the outbound logistics facility. Let the various lead times be denoted as T_1 ... T_7. We assume that these times are normally distributed with means greater than thrice respective standard deviations. We obtain

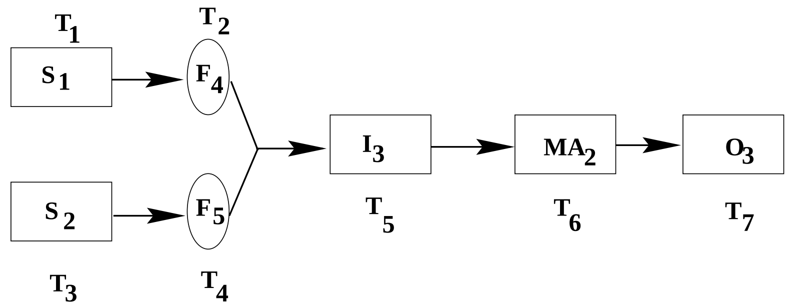

Figure 2: The reduced supply chain network for product C

the average supply chain lead time as $\max\{(T_1 + T_2), (T_3 + T_4)\} + T_5 + T_6 + T_7$. We know that the sum of any number of independent normal random variables is normal with additive means and variances. Also, to compute the maximum of two normal random variables, we use Clarke's results (see [3]), by assuming that the flow times at the various individual facilities are independent of each other. Thus, the total lead time is easily computed for the static and stochastic case. We cite below Clarke's results, for the maximum of two Gaussian random variables D_1 and D_2. The maximum is again assumed to be Gaussian (D). The maximum of any number of such random variables is easily derivable, thus resulting in a quick and effective method for performance modeling of even large supply chains.

$$\begin{aligned}
\mathbb{E}[D] &= \mathbb{E}[D_1]\Phi(\alpha) + \mathbb{E}[D_2]\Phi(-\alpha) + a\phi(\alpha) && (1)\\
\mathbb{E}[D^2] &= (\mathbb{E}^2[D_1] + \sigma_{D_1}^2)\Phi(\alpha) + (\mathbb{E}^2[D_2] + \sigma_{D_2}^2)\Phi(-\alpha) && (2)\\
&\quad + (\mathbb{E}[D_1] + \mathbb{E}[D_2])\ a\ \phi(\alpha) &&\\
\alpha &= \frac{1}{a}(\mathbb{E}[D_1] - \mathbb{E}[D_2]) && (3)\\
a^2 &= \sigma_{D_1}^2 + \sigma_{D_2}^2 - 2\sigma_{D_1}\sigma_{D_2}\varrho && (4)
\end{aligned}$$

In the above equations, ϕ is the standard normal density function and Φ is the corresponding cumulative distribution function. Also, ϱ is the coefficient of correlation between the variables D_1 and D_2, which is assumed to be 0 in our case, owing to the independence assumption.

The above result easily extends to the case where we have to evaluate the maximum of more than two normal random variables by observing that $\max\{A, B, C\} = \max\{\max\{A, B\}, C\}$.

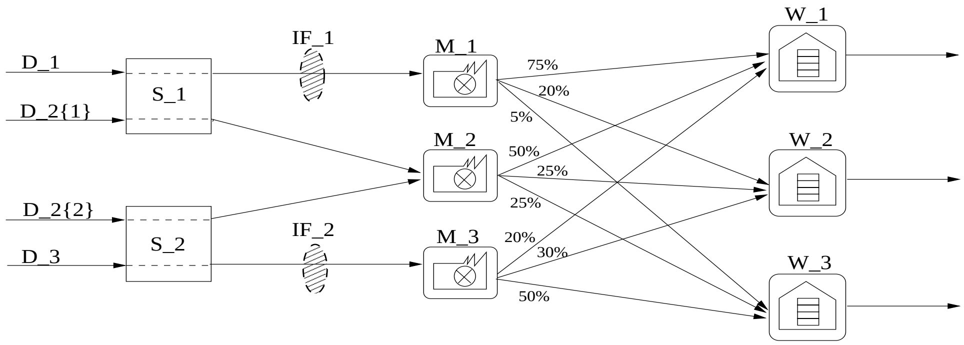

Figure 3: The supply chain for Example 1

5 Generalized Queueing Network Models

In this section, we present a method using general queueing networks, for evaluating the performance of make-to-order supply chains. This is the dynamic case wherein orders for end products arrive at random points of time, and demand a random processing time at each one of the facilities. Refer [2] for instances where business processes of an enterprise are modeled using queueing networks.

In this paper, we analyse the situation wherein three *different* manufacturers supply a common end product (for e.g. three different television producers) to a common market consisting of three distribution centres (DCs). We bring out the significance of alliances which appear as *interfaces* in our models.

5.1 The Single Product Multiple Competitors Case

Consider the network shown in Figure 3. Let us assume that there is a steady demand for products of a type, at the three warehouses W_1, W_2, and W_3. There are three different manufacturers M_1, M_2, and M_3 for this product, who supply to the same three markets. There are two common suppliers S_1 and S_2 with the alliances between them and the manufacturers as shown in the figure. We assume that M_2 has good alliances in place with both S_1 and S_2. Thus, we have interfaces between M_1 and S_1, and M_3 and S_2, which essentially slow down the process of supplying raw materials/sub-assemblies to the manufacturers. Each manufacturer shares the production with the warehouses as shown in the figure. The demand for the end products translate into demand for the required raw materials to the suppliers. We model the physics of the above supply chain as a generalized open queueing network of the single class type, for purposes of approximate analysis.

In Figure 4, we show the generalized queueing network (GQN) model for the above supply chain with input parameters as in Table 1.

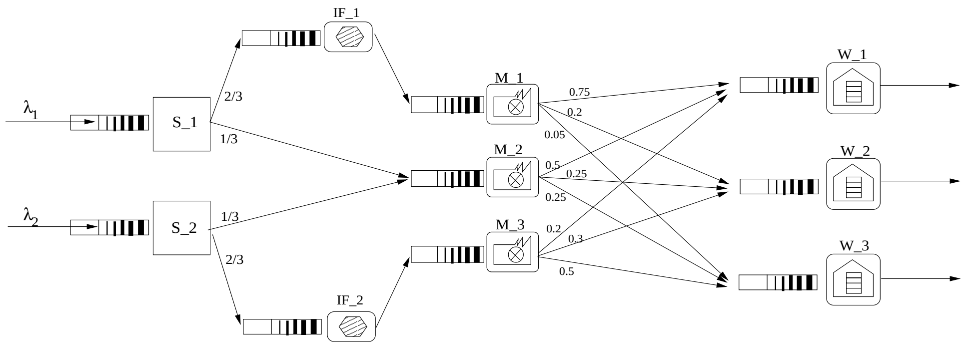

Figure 4: The generalized open queueing network model

$D_1 = D_2 = D_3 = D = 20$ units per hour; SCV=2.0 each. i.e. $\lambda_1 = \lambda_2 = 30$

Server Name	Service Rate in units/hour	
	Mean	SCV
S_1	40	1.5
S_2	40	1.0
IF_1	25	0.5
IF_2	25	0.5
M_1	45	1.5
M_2	50	1.0
M_3	45	0.5
W_1	60	0
W_2	60	0
W_3	60	0

Table 1: The input parameters for the supply chain of Figure 3

Manufacturer	Arrival Rates		
	D+D/2, D+D/2	D+0.05D, D+0.95D	D+D/4, D+3D/4
M_1	0.3629	1.1195	0.3509
M_2	0.1535	0.9078	0.1984
M_3	0.3313	0.3342	0.3882

Table 2: Effect of order release on average cycle times: base case

Manufacturer	Arrival Rates		
	D+D/2, D+D/2	D+0.05D, D+0.95D	D+D/4, D+3D/4
M_1	0.4509	1.2043	0.4388
M_2	0.1540	0.9084	0.1989
M_3	0.3316	0.3344	0.3884

Table 3: Effect of SCV of interface times on average cycle times, with $SCV_{IF_1} = 1.5, SCV_{IF_2} = 0.5$

Manufacturer	Arrival Rates		
	D+D/2, D+D/2	D+0.05D, D+0.95D	D+D/4, D+3D/4
M_1	0.4510	1.2043	0.4389
M_2	0.1541	0.9085	0.1991
M_3	0.3754	0.3772	0.4321

Table 4: Effect of SCV of interface times on average cycle times, with $SCV_{IF_1} = 1.5, SCV_{IF_2} = 1.0$

Manufacturer	Arrival Rates		
	D+D/2, D+D/2	D+0.05D, D+0.95D	D+D/4, D+3D/4
M_1	0.3188	1.0768	0.3068
M_2	0.1530	0.9074	0.1980
M_3	0.2886	0.2907	0.3445

Table 5: Effect of zero SCV of interface times on average cycle times

Manufacturer	Arrival Rates		
	D+D/2, D+D/2	D+0.05D, D+0.95D	D+D/4, D+3D/4
M_1	0.3110	0.8676	0.2909
M_2	0.1270	0.6670	0.1553
M_3	0.2810	0.2686	0.3267

Table 6: Effect of SCV of *arrivals* on average cycle times, with $SCV_1 = 1.0, SCV_2 = 1.0$

We model various facilities (suppliers, manufacturers, etc) as servers of the queueing network. Since we consider the single product type case, we *aggregate* the demand for all the three manufacturers as two independent arrival streams to the suppliers. The probability of routing the raw parts/sub-assemblies to the manufacturers (which is computed as a percentage of demand outsourced by the manufacturers to the suppliers) is depicted in the Figure 4.

For the examples presented here, we ran the GQN models using a package developed at the Laboratory for Competitive Manufacturing, Department of Computer Science, Indian Institute of Science [6], for analyzing the queueing models. This package makes use of results due to Whitt [10].

5.2 Discussion of Results

The performance measures of interest to us are the average cycle time and average total WIP at the three manufacturing plants. In the context of supply chain management, a manufacturer would usually be interested in knowing how much of his requirements he has to outsource from a given supplier. We studied three different scenarios for the above issue, with M_2 outsourcing different amounts of its demand from S_1 and S_2. This is modeled as different splitting rates of demand, thus influencing the arrival rates to the two suppliers. In the tables from Table 2, we considered three different such *outsourcing* combinations: one with equal splitting of M_2's demand among the suppliers, second with very little going to S_1, and the third a via-media case. The values for the average waiting time and average queue lengths have been tabulated for all the three cases. On critically observing the tables, we conclude as follows:

- For M_2, splitting demand equally, rewards it a lot in terms of reduced lead times and queue lengths (which turn out to be WIP in a manufacturing setting).

- If M_1 and M_3 decide on the amount to be outsourced from S_1 and S_2 without paying heed to what M_2 decides, it would prove sub-optimal to them. For example, M_1's optimal value occurs when M_2 decides to outsource 3/4th of its demand from S_2. But this won't happen, since, M_2 has decided to split demand equally!

  Thus before giving sub-contract to any of its suppliers, a manufacturer must decide keeping in view *all other customers of S_1 and S_2, who could very well turn out to be his own competitors*.

- The effect of interface time variance on the performance of M_1 and M_3 can be seen clearly from Tables 3 and 4. As the SCV (and hence, variance) of the interface IF_2 decreases from 1.0 to 0.0, (keeping the mean service rates at their base values), both the average lead time and the average inventory decrease. It is also seen, (see Tables 2, 3) that if the SCV of IF_1 is also reduced, in conjunction with IF_2, it will reduce the performance measures for M_1 and M_3. The best performance is obtained when the SCV's are zero.

- Let us now compare Table 2 with Table 6. For the former, the SCV of inter-arrival times are each 2.0, while for the latter, we decreased it to 1.0, each, fixing the mean inter-arrival times. We see that irrespective of the splitting pattern, the performance measures have improved. The average cycle times and inventories have reduced when we reduced the SCV of the arrival rate.

5.3 Implications

As discussed in the earlier section, our simple analysis (though approximated at a very aggregate level) brings out some very relevant issues in managing supply chains, unequivocally. They are:

1. Decisions by a manufacturer (retailer) regarding how much to be outsourced (bought) from which supplier (manufacturer), are not to be made in isolation. A thorough survey of the number, demand patterns and suppliers of the competing manufacturers (retailers) has to be made at the outset.

2. Although interfaces exist in the event of alliances not being proper, efforts must be put towards reducing the variance (and mean, if possible) of the interface process cycle times. Also, it would be better for M_1 and M_3 to get into alliance with the suppliers, like M_2 has got into, so that their interfaces are smoothened out.

3. The variance of arrival rate should be properly adjusted and controlled to yield better performance. Efforts aimed at reducing SCV of arrival rate (keeping its mean at the same value) yields better results.

6 Conclusions

In this paper, we have presented static and dynamic lead time models for analysing the supply chain process. The static models are based on the assumption that processing times are normally distributed. We used Clarke's approximation to get estimates of the mean and variance of the total supply chain lead time. This can be used in determining the delivery reliability for end customer orders. We have also proposed generalized open queueing network models for the dynamic performance evaluation of supply chain process. These are basically meant for make-to-order situations.

References

[1] B. C. Arntzen, G. G. Brown, and T. P. Harrison. Global supply chain management at Digital Equipment Corporation . *Interfaces*, 25(1):69–93, Jan-Feb 1995.

[2] J.A. Buzacott. Commonalities in reengineered business processes: Models and issues. *Management Science*, 42(5):768–782, May 1997.

[3] C. E. Clarke. The greatest of a finite set of random variables. *Operations Research,* pages 145–161, March-April 1961.

[4] M.A. Cohen and H.L. Lee. Strategic analysis of integrated production distribution systems: Models nad methods. *Operations Research,* 36(2):216–228, March-April 1988.

[5] D. Connors, C. An, S. Buckley, G. Feigin, A. Levas, N. Nayak, R. Petrakian, and R. Srinivasan. Dynamic modeling of re-engineering supply chains. Technical Report RC-19944, IBM Research Center, Almaden, 1995.

[6] V. K. Kumar. A package for performance analysis of generalized queueing networks. Technical report, Department of Computer Science and Automation, Indian Institute of Science, Bangalore, 1994.

[7] N. R. S. Raghavan and N. Viswanadham. Performance modeling and dynamic scheduling of make-to-order supply chains. In *Third International Conference on Agile and Computer Integrated Manufacturing, Renesslaer Polytechnic Institute*, October 1998.

[8] J. K. Sankaran and N. R. S. Raghavan. Locating and sizing plants to bottle Propane in South India. *Interfaces*, 27(6):1–15, Nov-Dec 1997.

[9] N. Viswanadham. *Analysis and Design of Manufacturing Enterprises*. Book to be published, 1999.

[10] W. Whitt. The queueing network analyzer. *Bell Systems Technical Journal*, 62(9):2779–2815, September 1983.