Design of Synchronized Supply Chains: A Six Sigma Tolerancing Approach

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Abstract

A supply chain network can be viewed as a network of facilities in which a customer order will flow through internal business processes such as procurement, production, and transportation, ultimately reaching the required products on time to customers. The delivery performance of such a network can be maximized by pushing the work through the system in a way that the finished products reach the customer in a customer specified delivery window, with a very high probability. This entails synchronization and hence strict control of variability of deliveries at all intermediate points, to ensure that the raw materials and semi-finished work arrive at work spots in a timely fashion. In this paper, we explore the use of the Motorola six sigma tolerancing methodology to achieve synchronization in supply chains. In particular, we use the six sigma approach to: (1) analyze a given supply chain process for six sigma delivery performance; and (2) design synchronized supply chains to guarantee six sigma delivery performance. We provide an example of a plastics industry supply chain, for which we report analysis and design experiments that demonstrate the use of the six sigma approach in designing synchronized supply chains with high levels of delivery performance.

1 Introduction

In the simplest sense, the supply chain represents a process of delivering value to customers by creating and delivering products. Supply chains span from raw materials to manufacturing, distribution, transportation, warehousing, and product sales. The end goal of supply chain coordination is synchronization, or each member acting in ways that are appropriately timed with the actions of other supply chain members.

In this paper, we are concerned with how synchronization among internal processes in a supply chain can be achieved and built into a supply chain network, leading to what we call a *synchronized supply chain*. Towards this end, we invoke ideas from statistical tolerancing in general and the Motorola six sigma approach in particular. We believe synchronization is the key to achieving outstanding delivery performance and customer service levels in a supply chain (that is, guaranteeing a high probability of delivery within the promised delivery window or what we may call customer tolerance interval). Synchronization in addition will lead to reduction of lead times and inventory levels, thus contributing to overall cost reduction.

1.1 Review of Relevant Work

Combating various sources of uncertainty in supply chains has been studied by a number of researchers. Some of the important references include [1, 2, 3]. The role of variability reduction as a means of lead time reduction is a popular topic in the area of manufacturing systems [4] and in the area of business processes such as new product design and development [5]. This paper is concerned with how variability reduction leads to better synchronization in a supply chain. Synchronization is also the key theme behind the Just-in-Time (JIT) philosophy in manufacturing.

We explore the use of mechanical design tolerancing approaches towards designing synchronized supply chains. A survey of tolerance analysis and synthesis approaches can be found in [6, 7, 8]. The Motorola six sigma approach is described in [9, 10].

1.2 Outline

In this paper, we explore the use of the Motorola six sigma approach to:

- analyze a given supply chain process for six sigma delivery performance; and

- design synchronized supply chains with six sigma delivery performance.

By six sigma delivery performance, we mean that the probability of delivering products to customers within a customer specified delivery window is at least as high as 0.9999966 [10].

In Section 2, we provide an overview of the Motorola six sigma approach to tolerancing. We bring out the notion of process capabilities and the meaning of six sigma performance. Section 3 presents an example of an aggregate-level supply chain in a plastics industry and we discuss analysis of lead time performance of this supply chain using six sigma terminology. In Section 4, we report on two different types of design studies on this supply chain. These are related to determining a pool of values for nominals and finding a variance pool.

2 Six Sigma Approach to Tolerancing: An Overview

Six sigma quality is the benchmark of excellence for product and process quality, popularized by Motorola [9, 10]. It provides a quantitative, statistical notion of quality useful in understanding, measuring, and reducing variation. A product is said to be of six sigma quality if there are no more than 3.4 non-conformities per million opportunities (3.4 ppm) at the part and process-step level, in the presence of typical sources of variation. The six sigma quality concept recognizes that variations are inevitable due to insufficient design margin, inadequate process control, imperfect parts, imperfect materials, fluctuations in environmental conditions, operator variations, etc.

Tolerance analysis and synthesis in the six sigma program are based on six sigma characterization of products and processes. The process capability indices C_p and C_{pk} are used as the vehicles to characterize the product-process quality.

Process Capability Indices

Let U and L be the upper and lower specification limits, respectively, of a part or subassembly dimension in the case of mechanical assemblies. In the case of a supply chain process, they represent the maximum and minimum lead times tolerated for an individual business process or the overall supply chain process. In the sequel of this section, we will explain the concepts with relevance to mechanical assembly design. Interpretation to the supply chain context is straightforward. Assume that σ is the standard deviation of the process that produces the dimension. Then, the index C_p is defined as:

$$C_p = \frac{U - L}{6\sigma}$$

The numerator above represents the specification width whereas the denominator captures the total width of the 3σ limits of the process distribution. For the rest of the discussion, assume that the process is normally distributed. The denominator then represents 99.73% limits for the process distribution. If $C_p = 1$, the implication is that the specification width is the same as the distribution width and when the process mean is centered at $\left(\frac{U+L}{2}\right)$ without any shift, the probability that the actual dimension is within the specification limits is 0.9973 (2700 ppm defect rate). Similarly, if $C_p = 2$, we have that the specification width is twice that of the distribution. In this case, when the process mean is centered at $\left(\frac{U+L}{2}\right)$ without any shift, the probability of conformance is 0.999999998 (.002 ppm defect rate). Since $\left(\frac{U-L}{2}\right)$ is the tolerance T of the part dimension (or in general of any attribute of a product), we have that:

$$\sigma = \frac{U - L}{6C_p} = \frac{T}{3C_p}$$

The index C_p does not capture how far away the process mean μ is from the ideal value τ (target value). The Motorola six sigma program assumes that the ideal value of the process mean is the midpoint of the specification interval, i.e. $\left(\frac{U+L}{2}\right)$. The index C_{pk} captures the effect of the shift in the process mean in the following way:

$$C_{pk} = C_p(1 - k) \quad \text{where} \quad k = \frac{|\tau - \mu|}{\left(\frac{U-L}{2}\right)}$$

The factor k above can be interpreted as the fraction of tolerance consumed by the mean shift. The above definition of C_{pk} assumes that $\tau = \frac{U+L}{2}$ and for a general definition, refer [9].

The Motorola convention is to use a one sided mean shift of 1.5σ. The one sided mean shift is perhaps motivated by common physical phenomena such as tool wear. If $C_p = 2$ and $C_{pk} = 1.5$ (mean shift consumes 25 percent of the tolerance range), the probability of conformance can be shown to be 0.9999966, which is equivalent to 3.4 ppm. Thus $C_p \geq 2$ and $C_{pk} \geq 1.5$ imply six sigma quality, assuming a 1.5σ one sided mean shift. $C_p \geq 1$ and $C_{pk} \geq 0.5$ refer to three sigma quality, assuming a 1.5σ one sided mean shift.

The Motorola program assumes a relationship of the form

$$Y = a_0 + a_1 X_1 + a_2 X_2 + \ldots + a_n X_n$$

If there is no mean shift, then the following formulae (called the Root Mean Square or RSS formulae) are applicable.

$$\begin{aligned} \mu_Y &= a_0 + a_1\mu_1 + a_2\mu_2 + \ldots + a_n\mu_n \\ \sigma_Y^2 &= a_1^2\sigma_1^2 + a_2^2\sigma_2^2 + \ldots + a_n^2\sigma_n^2 \end{aligned}$$

Recall that σ_i, for $i = 1, \ldots, n$, can also be written as:

$$\sigma_i = \frac{T_i}{3C_{pi}}$$

where T_i is the tolerance range of the ith part and C_{pi} is the C_p value for the ith part ($i = 1, \ldots, n$). In the presence of a mean shift, the six sigma program suggests the use of the Dynamic RSS method, where the C_{pk} values, $C_{pk1}, \ldots, C_{pkn}$, of the individual processes corresponding to dimensions $X_1, \ldots, X_n$; and the tolerances, $T_1, \ldots, T_n$, of the individual parts, are used in the following way to compute the variance of Y:

$$\sigma_Y^2 = a_1^2 \left(\frac{T_1}{3C_{pk1}}\right)^2 + \ldots + a_n^2 \left(\frac{T_n}{3C_{pkn}}\right)^2$$

Note that the standard deviations σ_i are inflated by an amount equal to $\frac{C_{pi}}{C_{pki}}$, for $i = 1, \ldots, n$. Thus the dynamic RSS method emulates random behavior in the process mean by inflating the process standard deviation.

Tolerance synthesis employs the approach of using tolerance analysis in an iterative way. Each iteration will evaluate the resulting probability of non-conformance and the C_p and C_{pk} values. The synthesis procedure seeks to obtain a probability of non-conformance of at most 3.4 ppm, which is guaranteed by $C_p \geq 2$ and $C_{pk} \geq 1.5$. The synthesis can assume several forms: finding optimal values for nominal dimensions; finding optimal values for tolerances; and establishing a variance pool that can be allocated to individual processes so as to obtain the desired assembly yield.

3 An Example

We now consider a supply chain for a plastics industry (a certain anonymous firm in the western state of Maharashtra, India) and provide the basis and notation for applying statistical tolerancing concepts. The supply chain process in question has six business processes:

1. **Procurement**: The chemicals and other raw materials that are used in the manufacturing of the plastic are procured in this stage. Typically, several suppliers are involved. At an aggregate level, we will assume one mega supplier. Let X_1 denote the procurement lead time.

2. **Sheet Fabrication**: Here, from the chemicals and other raw materials, thin sheets of plastic are fabricated at a fabrication plant. These sheets are then transported to another plant for manufacturing and assembly. Let X_2 be the sheet fabrication lead time.

3. **Transportation**: The thin plastic sheets fabricated in the earlier stage are transported in trucks to a manufacturing and assembly facility. Assume that the logistics lead time here is X_3.

4. **Manufacturing**: In the manufacturing and assembly plant, numerous types of components are manufactured from the plastic sheets. This is a multistage process and the processing depends on component types. We will aggregate this business process into a single stage and call the manufacturing lead time as X_4.

5. **Assembly**: In the same manufacturing and assembly facility, the components produced are assembled into different types of customer-desired products. Customization also happens here. We assume the assembly lead time to be X_5.

6. **Delivery**: Warehouses are located in the manufacturing and assembly plant itself. The final products are stacked here and supplied directly to customers against their orders. Third party logistics providers are used here. Let X_6 be the delivery time to a certain pool of customers who are co-located.

For the above system, we postulate that JIT philosophy is used and there is negligible waiting between one business process and the next one. The supply chain process lead time, Y, can then be described as:

$$Y = \sum_{i=1}^{6} X_i$$

Note that X_is ($i = 1, \ldots, 6$) are mutually independent continuous random variables and hence Y is also a continuous random variable. Furthermore, if X_is are normally distributed, then Y is also normally distributed.

Table 1: Notation for different measures ($i = 1, \ldots, 6$)

BP_i	Business process i
X_i	Lead time of BP_i
τ_i	Target value or nominal of X_i
T_i	Tolerance of X_i
μ_i	mean lead time produced by BP_i
σ_i	standard deviation of lead time X_i
U_i	Upper Specification limit of X_i
L_i	Lower Specification limit of X_i
(L_i, U_i)	Delivery window for BP_i
C_{pi}	C_p index for the process BP_i
C_{pki}	C_{pk} index for the process BP_i
Y	Supply chain process lead time $= \sum_{i=1}^{i=6} X_i$
τ_Y	Nominal or target value of Y
T_Y	Tolerance of Y
μ_Y	mean of Y
σ_Y	standard deviation of Y
U_Y	Upper Specification limit of Y
L_Y	Lower Specification limit of Y
(L_Y, U_Y)	Delivery window for the supply chain process
C_{pY}	C_p index for the supply chain process
C_{pkY}	C_{pk} index for the supply chain process

Table 1 shows the notation we will use in the rest of the paper. Table 2 shows the typical nominals, tolerances, and standard deviations for the X_is. The standard deviations have been computed from the tolerances by assuming $C_p = 1$ (three sigma performance) for each individual process. The standard deviation values will be halved when we assume $C_p = 2$. Using the RSS formula, the standard deviation σ_Y of the supply chain process can be computed easily as the square root of the sum of squares. In this case, $\sigma_Y = 2.135$ days. Table 3 shows the specification range, tolerance, and the tolerance interval for Y for different specified values of C_p and C_{pk}. They are computed using the following formulae:

$$U_Y - L_Y = 6\sigma_Y C_{pY}$$

$$T_Y = \frac{U_Y - L_Y}{2}$$

$$\text{Tolerance Interval} = (\tau_Y - T_Y, \tau_Y + T_Y)$$

We have chosen $\tau_Y = 83$ days, which is simply the sum of the nominals of the cycle times of the six business processes.

From Table 3, it is clear that for the given supply chain process to be six sigma, the specification range should

Table 2: Nominals, tolerances, and standard deviations (all values in Days)

	Lead Time	τ_i	T_i	σ_i
X_1	Procurement	7	1	0.334
X_2	Sheet fabrication	30	3	1.0
X_3	Transportation	3	1	0.334
X_4	Manufacturing	30	5	1.667
X_5	Assembly	10	2	0.667
X_6	Delivery	3	1	0.334

Table 3: Tolerances and specification ranges for Y (all values in Days)

(C_{pY}, C_{pkY})	$U_Y - L_Y$	T_Y	Window
(0.5, 0)	6.405	3.2025	(79.8, 86.2)
(1.0, 0.5)	12.81	6.405	(76.6, 89.4)
(1.5, .09375)	19.215	9.61	(73.4, 92.6)
(2.0, 1.5)	25.62	12.81	(70.2, 95.8)

be very wide, namely (70.2, 95.8). That is if the customers are prepared to tolerate such a wide window for delivery, then the process becomes six sigma even though the individual processes have been assumed to be three sigma processes. If the customers' tolerance interval is (76.6, 89.4), then the supply chain process becomes a three sigma process. If for the same tolerance range, the supply chain process is to be six sigma, then it would imply that the standard deviations of individual processes should be cut to half their original values, which means the individual processes themselves should be six sigma.

4 Design Optimization for Six Sigma Performance

Here, we describe two different types of design experiments. In the first, we determine the range of nominal values for the lead times of designated internal business processes, so as to achieve six sigma delivery performance. In the second, we compute a variance pool for lead times of designated business processes (hence their process capabilities) to achieve six sigma delivery performance.

4.1 Finding a Nominal Pool

Table 4 shows the structure of this design problem. In this problem, the tolerance T_Y of the supply chain process is given. The tolerances for the lead times of individual processes are given as also the nominals of lead times of some of the business processes (say, τ_2, τ_4, τ_5). The problem is to compute a range of values for the other nominals (in this case, τ_1, τ_3, τ_6), so as to achieve six sigma performance. As an example, let us say we know the nominals of the fabrication lead time, manufacturing lead time, and assembly lead time; and we are to obtain a range of values of nominals for the procurement lead time, transportation lead time, and the logistics lead time. The first of these has implications on choice of suppliers, while the second and third influence the choice of logistics providers and location of suppliers. We can solve the problem through the following algorithm:

1. Assume appropriate process capabilities for individual business processes. Since we know the tolerances T_i, we can compute the standard deviations σ_i using the RSS method or the dynamic RSS method.

2. Compute σ_Y as the root of the sum of squares of σ_i's.

3. Knowing T_Y and σ_Y, compute the range of values of nominals in the set $A-B$ over which six sigma delivery performance is guaranteed.

4. The pool of nominal values obtained in the previous step can be distributed to individual business processes based on engineering judgment and any other available information.

For example, let $\tau_Y = 83$ days; $T_Y = 6$ days; $T_1 = 1$; $T_2 = 3$; $T_3 = 1$; $T_4 = 3$; $T_5 = 2$; $T_6 = 1$. Let the nominal value of fabrication lead time (τ_2) be known to be 30 days; that of manufacturing lead time (τ_4) to be 30 days; and that of assembly lead time (τ_5) to be 10 days. It is required to find a range of values for the pool of other nominals $\tau_1 + \tau_3 + \tau_6$ such that the probability of delivery is at least 0.9999966 within the delivery window. To solve this problem, we first assume that all the individual processes are six sigma and compute individual standard deviations. To be conservative and realistic, we assume shifts and drifts in the mean ($1.5\sigma_i$), thus we use $C_{pki} = 1.5$ to compute

Table 4: Finding a nominal pool

Given:

1. τ_Y, the target value for Y
2. T_Y , the tolerance range for Y
3. T_i $(i = 1, \ldots, 6)$
4. A subset B of $A = \{\tau_1, \ldots, \tau_6\}$

To Compute:
Range of values for all nominals in the set $A - B$ over which the supply chain process exhibits six sigma delivery performance

the standard deviations. Then we compute σ_Y. It is found that as long as $\tau_1 + \tau_3 + \tau_6$ is in the range (12.5, 13.5), we obtain six sigma delivery performance. That is, for this range of values, the probability of Y to be in the range (77, 89) is at least .9999966. The maximum probability is attained at 13.0 days. We can choose any value that is convenient in this range. For example, the value 13.5 days provides us the most flexibility. This can be allocated to τ_1, τ_3, and τ_6 in any possible way, based on best engineering judgment and any other technical considerations.

4.2 Finding a Variance Pool

Table 5 shows an outline of this problem. In this problem, the nominal value of the overall process, τ_Y, as also the nominal values for the individual business processes are given. The tolerance T_Y of the overall process and also that of some individual business processes are given. The problem is to find a variance pool that can be distributed across the individual business processes. How we distribute the variance pool can depend on our knowledge of the processes and best engineering judgment. This problem has implications on choice of suppliers, logistics providers, machining equipment, etc. For example, let the target values be: $\tau_Y = 83$ days; $\tau_1 = 7$; $\tau_2 = 30$; $\tau_3 = 3$; $\tau_4 = 30$; $\tau_5 = 10$; $\tau_6 = 3$. Also, assume that $T_2 = 3$; $T_4 = 3$; $T_5 = 2$. If it is known that the above three processes (fabrication, manufacturing, and assembly) are six sigma, then the standard deviations can be computed by using the value of C_{pk} which in this case is 1.5. This will take into account shifts and drifts in the mean value of the processes (Dynamic

Table 5: Finding a variance pool

Given:

1. τ_Y , the nominal value of Y
2. T_Y , the tolerance of Y
3. A subset B of $A = \{T_1, \ldots, T_6\}$
4. τ_i $(i = 1, \ldots, 6)$, the nominal values of X_i

To Compute:
A variance pool for standard deviations of business processes in the set $A - B$.

RSS). This leads to:

$$\sigma_i = \frac{T_i}{4.5}; \quad i = 2, 4, 5$$

Now assume T_Y to be 6 days. If we require that the the delivery performance be six sigma, we need $C_{pY} = 2.0$ and $C_{pkY} = 1.5$. Using this, we can compute the standard deviation σ_Y for the supply chain process lead time. We will then get the variance pool, ${\sigma_1}^2 + {\sigma_3}^2 + {\sigma_6}^2$, for the three processes, viz. procurement, transportation, and delivery as 0.32. This variance pool can then be distributed across the individual processes in any way guided by engineering judgment. For different values of T_Y, we will get different variance pools. For example, if $T_Y = 7$ days, then the variance pool is 1.01; if $T_Y = 8$ days, then the variance pool is 2.11; and so on.

5 Conclusions

Synchronization among internal business processes of a supply chain is a key to achieving a high level of delivery performance in supply chains. In this paper we have shown how the Motorola six sigma approach can be used in designing synchronized supply chains. We believe the paper is an important contribution towards applying statistical tolerancing techniques and best practices to design supply chain networks with high levels of delivery performance. There is available a rich variety of statistical tolerancing methodologies and best practices [8] and a natural direction for further work is to look into the application of those in designing synchronized supply chains.

In this article, we have only described (in Section 4) two possible design experiments. There are a rich variety of such design experiments we can conduct using the six sigma approach and other statistical tolerancing approaches and best practices. Also, we have implicitly assumed all distributions to be normal. Further the case study has been addressed at a coarse level of detail. There is available a rich variety of more general statistical tolerancing methodologies and best practices [8] and a natural direction for further work is to look into the application of those in designing synchronized supply chains.

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